

Use of a numerical strategy framework in the professional development of teachers

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Abstract: Derived initially from a strategic analysis of children's methods of counting, the New Zealand Numeracy Projects used, as a starting point for the professional development of teachers, a strategy framework that traces children's development in number reasoning. A pilot study indicated the usefulness of professional development where teachers use the framework to determine the number reasoning of students in their own classes. Subsequently, as part of the professional development offered for the Projects, a DVD showing numerous video clips of students was produced to show teachers what range of number strategic thinking they might expect in their classes. In the next five years more clips were added and some edited out. This paper outlines how the video clips were incorporated into the initial stages of the enhanced teacher professional development model to enhance teaching effectiveness in using the strategic number framework, and how these clips are used in the pre-service education of student teachers at the University of Auckland.

Introduction

Arising originally from disappointing results in the Third International Mathematics and Science Study (Garden 1996, 1997) the New Zealand Ministry of Education delivered a series of the Numeracy Projects (NPs) that covered students in years 1 to 10; the NPs began in 2000 and ended in 2010.

In 2000 a Count Me In Too (CMIT) pilot, which was a Professional Development (PD) project for year one to three students (NSWDET, 1999), was trialled (Thomas & Ward, 2001). In 2001 this was succeeded by the Early Numeracy Project (ENP) (Thomas and Ward, 2002) for the same year groups. Also in 2001 the Advanced Numeracy Project (ANP), which was for PD of teachers of year 4 to 6 students (Higgins, 2002), was introduced. In 2002 a PD project, the Intermediate Numeracy Project (INP) was added (Irwin, 2003) - in New Zealand intermediate schools operate at the middle school level. Also in 2002 a PD project, Te Poutama Tau, delivered in the Maori language was developed for teachers working in Maori language immersion classes (Christensen, 2003). Finally, in 2005, the Secondary Numeracy Project (SNP) for PD of mathematics teachers teaching students in the first two years of secondary school was introduced (MoE, 2006) - these are years 9 and 10 in the New Zealand school system.

The intention of this set of nation-wide projects was to give as many primary teachers and secondary mathematics teachers the opportunity for PD in teaching numeracy. Logistically it proved impossible to train enough numeracy PD facilitators for the projects for them to be delivered in a short period of time; consequently the projects took eleven years to deliver. About 27 000 primary teachers, representing over 95% of the teaching force, participated in the projects; performance data from the approximately 540 000 students they taught was gathered by the Ministry of Education. Only some secondary mathematics had the opportunity to participate in the SNP.

At the core of the PD of all the NPs was an attempt to shift teachers' pedagogy to take into account the strategic number thinking of their students. A small scale trial before the start of the projects (Hughes, 1995) indicated that New Zealand teachers knew very little about the complexities of their students' numerical strategic thinking, and that consequently their teaching tended to emphasise low-level algorithmic activity. The research was based on the work of Steffe and colleagues (Steffe et al, 1983, 1988) on children's stages in developing counting strategies, which was capped off with a part-whole reasoning stage where students changed from counting methods moving parts of numbers mentally to calculate answers. For example, a counter might solve $27 + 8$ by counting-on from 28 to 35 whereas a part-whole thinker would typically likely add 3 to get 30 then add a further 5 to get 35.

These strategy stages were spelt out in tabular form by Steffe's PhD student Bob Wright (Wright, 1991, 1998, Wright et al 2000, 2002). This framework was used by Wright as the basis for a Mathematics Recovery programme modelled on a New Zealand PD initiative called Reading Recovery (Watson & Agnew, 2009). In 1996 Wright was asked by the New South Wales Department of Education and Training (NSWDET) to create a strategic number thinking assessment tool for the CMIT project (NSWDET, 1999). This material was used in New Zealand in the CMIT pilot in 2000, and modified for use in the NPs in future years.

Description of a preliminary study to enhance professional development

By the early 1980s, the predominant "normal science" in mathematics education research, in which carefully gathered experimental data was interpreted through statistical tests like the null hypothesis from which

empirically based generalisations might be drawn, radical constructivists had developed a new paradigm where the teacher/researcher taught children and observed the children's mathematical actions (Steffe and Kieren, 1994). The teacher/researcher then used a conceptual analysis to build models of children's mathematical knowledge and its construction. This new paradigm served as the basis for the design of the enhanced professional development framework aimed at helping teachers to critically reflect upon and change their own mathematics teaching practices. An early example of this research was Children's Counting Types (Steffe et al, 1983); the types of children's number thinking would become embedded in the NPs.

There was evidence, before the start of the NPs in 2000, that the effective use by teachers of the strategy framework leading to part-whole thinking would require a shift in their view of teaching mathematics. As part of the enhanced professional development (PD) efforts, a pilot study was undertaken to examine this with three experienced teachers of year four students (Hughes, 1995). One of its objectives was to find the effect on teachers' practices through their diagnostic assessment of the counting types and part-whole thinking their children used. The study sought to find the effects on teaching by facilitating teachers' observing and finding out through individual interviews more about how children solve numerical problems.

Before the initial study it was conjectured that interviewing children from a teacher's class about their mathematical knowledge would be a disequilibrating factor in the teacher's pedagogy. The first author of this paper conjectured that teachers interviewing children in their classes about the number strategies mathematical would often surprise them as the teachers would typically lack understanding of their students' strategising; the difference in the mathematical processes actually used by children in their class and the teachers' belief about their processes would likely be significantly in conflict. A further conjecture was that, should disequilibrium occur, teachers would feel responsible for changing their teaching practice, and therefore would be more ready to experiment with teaching strategies that would enable them to accomplish this. For the sake of brevity, only one of the case studies involving a single teacher is reported in this paper. Her reactions were similar to the other two teachers who participated in this study.

Case study approach

A case study is a suitable research methodology when a holistic, in-depth investigation is needed (Feagin, Orum, & Sjoberg, 1991). Case studies are designed to bring out the details from the viewpoint of the participants by using multiple sources of data. Selected cases must be done so as to maximize what can be learned in the period of time available for the study. The participant teachers who formed the case studies of this study provided the reflective voices in understanding the impact of reviewing student interviews on learning efficacy upon classroom teaching practices.

The case study

Before interviewing three children in her class of differing abilities Ms B believed that the strategy stage that children operated at for addition was not important:

It is going to be faster [to use part-whole strategies rather than counting] but it doesn't matter. The way I think if they are using the fingers now, eventually they are not going to, there is no point in me saying you can't use your fingers.

However, after interviewing three children from her class and reviewing transcripts of year 4 children's strategic thinking in number from another class, Ms B was very surprised by their thinking:

I was overwhelmed at the variance of thinking of less able and more able children. I was not aware of the strategies [for single digit additions] that could be taught. In the past I would teach $6 + 5 = 11$. That is the answer and the children would eventually learn it. Now I see the importance of strategies. I can now see that

$$6 + 5 = 12 - 1 = 11 \text{ or } 6 + 5 = 10 + 1 = 11.$$

It appears that the children who can use strategies think laterally and this knowledge is transferred across all areas of maths and other curriculum.

Initially the children's' responses in the interviews of pupils in her class were confusing for the teacher:

I must be honest. I was lost at times. Only now can I understand their logic.

For example previously for $51 - 29 = 38$. I would not have analysed the problem. It was just right or wrong. I wondered could I ever make a difference with these children.

However, by the time she interviewed three more children from her class without the researcher present, her confidence had improved:

I was more knowledgeable. I could identify gaps more easily. The [part-whole] strategies [for single digit addition] I had taught were starting to appear. Halleluia!

Following the sessions where Ms B watched children in her class struggling to learn the standard arithmetic algorithms for subtraction, she began to criticise the way she was teaching it. She describes what she used to do:

To do $81 - 29$ in the vertical form. They would sit in a half circle on the mat with those little chalk boards, the place-value blocks and then I would put up 81 take away 29. Then straight away I used to do the crossing out with the chalk.

These teaching sessions led Ms B to re-evaluate her beliefs about teaching mathematics.

I've always thought children learnt progressively. And I've always thought that we teach progressively, that we teach one thing. For example, we teach no re-naming and then we teach re-naming double-digit. Then we teach no re-naming three-digits and then re-naming three digits like that, it was all sequential and it was progressive. But I know now after doing this [study] for the year that learning isn't progressive and teaching isn't progressive.

The interviews with children from her class proved to be an important element in stimulating Ms B to change her teaching methods in mathematics. Initially the way in which the children were thinking about mathematics was a matter of indifference, then a source of confusion and then surprise. A combination of pedagogical content sessions with the other teachers, which Ms B found challenging, and teaching sessions with students from her class, in which constructivist teaching was used, led Ms B to experiment with mathematics teaching methods that were much more child centred. Ms B was positive in her own mind that the mathematics learning that was now taking place in her class was superior to what had taken place before the study. The interviews with children from her class proved to be an important element in stimulating Ms B to change her teaching methods in mathematics.

The conjecture that exposing teachers to the mathematical thinking used by children in their own classes would lead those teachers to re-evaluate and experiment with their teaching proved true in the case of Ms B.

Production and use of video clips in the NPs

Encouraged by the evidence from this small-scale pilot, in 1998 the first author of this paper developed and taught a graduate diploma paper at Auckland College of Education that began with teachers learning about students' numerical strategy stages through viewing video clips from a CMIT video (NSWDET, 1997); this was followed up by teachers interviewing children in their classes using the CMIT diagnostic tool. Essentially this tool, with modifications, became the basis for the assessments of students' strategic thinking for enhanced PD of teachers in the NPs.

In the CMIT video some academics and teachers talk about CMIT, and a number of children solve number problem I a way that demonstrates their level of strategic thinking. Believing that teachers would learn more from seeing children solving problems than hearing the project being discussed the first author did not show the discussions between academics and the teacher studio audience, but did show the children solving number problems; this feature would continue in the video clips that the first researcher would make for the NPs. The mathematics curriculum officer for the Ministry of Education attended the lecture in one of the courses in which CMIT video clips were shown and discussed. Both the lecturer and curriculum officer felt that participant teachers were keenly interested in the clips. Consequently a decision to gather video clips of New Zealand children answering strategy questions as part of the 2000 CMIT pilot was made. More clips were gathered over the next six years. Eventually these clips included students from year 1 to year 10, and from stage 0 to stage 8 of the numeracy strategy framework (Table 1). By the end there were seven editions of the video clips DVD (Hughes et al, 2000 to 2006).

Table 1
Counting Stages and Part-whole Stages - Video clips by frequency and range of year range produced for the final DVD

| Stage | Counting Strategy Stages | Number of Clips | Student Year Level |
|-----------------------------------|------------------------------------|-----------------|--------------------|
| 0 | Emergent | 2 | Years 0-1 |
| 1 | One-to-one Counting | 5 | Years 1-2 |
| 2 | Counting from One on Materials | 3 | Years 0-2 |
| 3 | Counting from One by Imaging | 3 | Years 0-1 |
| 4 | Advanced Counting | 7 | Years 2-10 |
| Part-Whole Strategy Stages | | | |
| 5 | Early Additive Part-Whole | 14 | Years 1-10 |
| 6 | Advanced Additive Part-Whole | 9 | Years 3-10 |
| 7 | Advanced Multiplicative Part-Whole | 10 | Years 4- 10 |
| 8 | Advanced Proportional Part-Whole | 9 | Years 6-10 |

Note: In New Zealand schools year 0 students are those who have just recently started school.

Over time the writer/director of the videos, who is the first author of this paper, developed a number of principles for the production and selection of video clips for teacher PD:

- Only a small number of strategy questions would be asked of any student - no student would do a full whole interview.
- Students were not permitted to write down anything. This is designed to reduce the chance that students would try to use standard algorithms when they are expected to use strategic part-whole reasoning. (Algorithmic methods were deemed not to provide any evidence of students' strategic thinking.)
- The interviewers were not permitted to teach, rather they were expected to encourage the student explaining their thinking as best they could. Interviewers could ask clarifying questions like "What were you thinking?" but not offer any scaffolding. Clips where the interviewers, who were variously teachers and mathematics education lecturers, engaged in teaching by "leading the witness" to the answer were edited out.
- The clarity of student actions and words indicating their strategic method was important. Because the video clips would normally be the first exposure of teachers to the strategies framework undertaking the PD special care was taken to use clips where the student explanation or actions made it plain what strategy they were using.
- Samples from a wide a range of ages for any given strategy stage were obtained. The intention was to indicate in the teacher PD that age is an unreliable predictor for the level of strategic teaching.
- Four students from one school were followed longitudinally over four years. The purpose was to indicate to teachers in the PD how student progress may occur through the framework. And indeed the progress was often striking; in one case the clips show a boy who in year 1 is at stage 1, in year 3 at stage 5, and in year five stage 6.
- The video clips were representative of New Zealand society. Students across all ages and stages were a balance of ethnicities and cultures in the New Zealand population. At any age it was unacceptable to have a predominance of any ethnicities in the lower stages and other ethnicities in the higher stages.
- Clips showing non-verbal clues to the strategy the student was such as length of time to answer, eye movements when strategising, and sub-vocalised counting were selected as important examples of what teachers should look for when interviewing their own students.

A powerful example of the importance of teachers looking for non-verbal clues is a year 10 girl whose verbal explanation appears to indicate she is using a Stage 5 Early Part-Whole method. To work out $47 + 8$ the teacher shows her the problem written on a card and reads it:

You have \$47 in the bank. You deposit \$8. How much money do you have in the bank now?

The student says the answer is \$55 within a second of the reading ending. Asked to explain she says:

I added 4. I just halved eight and just added each half onto the 47.

A cursory analysis of her thinking might indicate that this student was using part-whole thinking since she has split the eight into 4 and 4. Yet a sensible strategic part-whole split would be $8 = 3 + 5$, because $47 + 3 = 50$ and $50 + 5 = 55$. The $4 + 4$ split makes little sense indicating that the student might not have used a part-whole method at all. A close re-examination of the video clip shows that this is correct; while the teacher was not observing the student when starting to read the card the student immediately began moving her lips indicating that she was counting. A very plausible explanation of the student's method is that she was counting-on in two groups of four indicating she is at Stage 4 Advanced Counting (Table 1).

Strict protocols surrounded the creation then the use of the video clips. Parents, who gave written permission to video their children, were promised that the only people who could show the videos were facilitators for the NPs and mathematics education lecturers from the country's six universities. This was to assuage any understandable fears that that clips of their children might end up being shown on the Net.

The use of the video clips in professional development of teachers

Facilitators in the NPs had copies of the DVD that contained all the video clips. How they might use the DVD with teachers was variable across the six regions in which the NPs were delivered; due to this variability no attempt was made to assess the effectiveness of the use of the clips. However, some evidence was available from the use of the clips with school principals.

In response to perceived failures in the education system in New Zealand in 1989 the Prime Minister led a radical devolution of decision-making power from the then Department of Education (DoE) to individual schools (Lange, 1989). Parent-elected Boards of Trustees (BOTs), which had principals as members ex-officio, now had much of the power that the regional Boards of Education and the DoE previously had. In the capital, Wellington, the DoE was replaced by a much slimmer Ministry of Education (MoE) that had much reduced powers.

This devolution of power for the bureaucracy to the BOTs meant that any PD designed by the Ministry, as was the case in the NPs, could not be imposed on schools; the BOTs would decide whether or not their school would be involved. Of course principals are crucial in any decisions over involvement in PD as BOTs normally accept principals' recommendations for PD for their staffs.

As part of the delivery of the INP groups of intermediate principals were brought together by the MoE for a two-day course. The essential feature of the NPs, namely the strategy framework was introduced to the principals using the video clips.

One principal, who was a member of the NP Reference Group, which met twice a year to review progress and make recommendations for future PD developments, noted this PD day had a significant effect on his decision for his school to be involved with the NPs:

While I was well disposed to the Numeracy Projects through my contact in the Reference Group it was this day that changed me from taking a benign interest to want positively to involve my school in the Numeracy Projects.

Watching the video clips was intriguing and interesting:

The video certainly kept my attention - but it would be wrong to say that I wanted my school to be involved just by watching the video - though, on reflection it was a significant part of a process.

Students were brought in from a local intermediate school. Watching a NP facilitator interviewing these students about their strategic thinking and then engaging in conversations after the students had left had an important effect: We had the diagnostic assessment sheets, and we discussed which assessment sheet to use, and then what stage the students were at. I certainly was involved and interested - it had the ring of reality about it.

Principals were paired up in the afternoon and took turns at interviewing students:

I found the interviews very interesting. It reminded me of the reality of the classroom for my teachers. By the day's end I wanted my school in on the project.

Use of the Video Clips in Pre-service Teacher Education

Learning the mathematics pedagogy that is implicit in the NPs became a central part of mathematics education pre-service courses taught at the University of Auckland. The video clips are used in two ways:

- To help improve student-teachers' personal mathematical knowledge;
- To help student-teachers' learn how to conduct interviews in school to assess the strategic stage students were operating

The videos have had an important part in changing student-teachers view of the nature of mathematics. In particular the courses help to wean many of them off the algorithmic thinking they learned at school, and replace by using strategic mental processes. To this end student-teachers are shown video clips in lectures, then asked to explain how students solved the problems. For example, a video clip shows a year 7 student solving $403 - 97$, which is written on a card and read to him:

A boy has \$403 in his bank account. He takes out \$97 to buy a new skateboard. How much does he have left? Immediately the student says \$306. He explains his reasoning:

- You go 90 minus 400 is 310;
- Then you would minus by 7, which is 303;
- And then you have got this little 3, and you plus it

Student-teachers are then asked to discuss this solution method – often many of them are initially non-plussed by the strangeness of student methods. They then discuss using better solution methods.

For example $403 - 97$ can be more efficiently calculated mentally with these steps:

- $403 - 100 = 303$
- $97 + 3 = 100$
- $303 + 3 = 306$

In learning such mental techniques the student-teachers are encouraged to use materials as part of improving their own strategic methods; in this case a number line is ideal.

At the end of all courses students are assessed on their strategic ability in solving number problems. This is a high stakes assessment as failure means failing the whole course. This indicates the seriousness with which the University takes the issue of sending teachers into schools with sufficient content knowledge to teach mathematics well. Normally a content assessment in a course has 25 questions. This is typical question:

341 - = 299. What is the number that goes in the box? Explain your method

The correct answer with a correct explanation is awarded 1 mark. There are no half marks because they have been instructed not to use an explanation involving algorithms. So here the right answer, 42, with the use of an algorithm receives zero marks. However there a number of acceptable answers; the proviso is that it must be a part-whole method. For example a student-teacher might write:

- I transformed the equation to working out 341 minus 299
- I added 1 and 41 to get 42.

Simultaneously with the development of student-teachers mathematical content knowledge they interview students in schools to determine the stage of their strategic thinking (Table 1). To assist in learning to do this student-teachers are shown numerous video clips and asked to discuss what stage of strategic reasoning the students are using. Later student-teachers then report on their findings about the students that they interviewed.

This is followed by there is a high stakes assessment of students thinking involves showing four video clips that they have not seen previously. Here is typical question with model correct answers added in brackets:

The lecturer clicks the video on the Stage 5 clips, then selects Brian. The question reads: Write down notes on what Brian did and said.

(A typical Answer: The teacher says: How much is ninety-nine plus four more.

Brian, who in year 1, says: a hundred and three

The teacher says: How did you work that out so quickly?

Brian says: Because there is ninety-nine and four. I added on of a hundred and three for the hundred and three.)

The next question reads:

Question 1

Why is Brian at least Stage 5 "Early Additive Part-Whole Thinking"? Justify your answer.

(Typical answer: Brian broke 4 in to parts 3 and 1. He added the part 99 and 1 to give 100. He knew he had 3 more to add to this 100. According to the framework this is beyond counting. So he is at least stage 5. He may be stage six but this would require a more challenging question like $67 + 67$ to determine this.)

The next question reads:

Question 2

Beth works out $7 + 9$ is 16 very quickly. List three ways that she may have solved the problem using early part-whole (stage 5) methods.

(Answer:

Method 1:

$7 + 9$ is the same as $8 + 8$ by adding and subtracting 1. Beth instantly recalls $8 + 8 = 16$.)

Method 2

$7 + 9$ is the same as $7 + 10 - 1$ which is $17 - 1 = 16$.)

Method 3

$7 + 9$ is the same as $9 + 9 - 2$ which is $18 - 2 = 16$. Beth instantly recalls $9 + 9 = 18$.)

Two types of answers were marked wrong. A number of student-teachers say Beth instantly recalled the answer to $7 + 9$, but this receives no marks as it is not part-whole method. Some said Beth could have counted on from 10 to 17; this was excluded on two grounds, it is not a part-whole method, and it cannot be done very quickly.

The next question reads:

Question 4

Another student, Josephine, is asked what $9 + 8$ equals. She says 17 very quickly. Does this show that she is an early part-whole thinker? Explain.

(Answer: Due to the answer 17 being given quickly there are two realistic possibilities namely instant recall, which is nothing to do with strategic thinking, or an early part-whole method.)

While this was not asked in this question student-teachers have been taught to ask new questions like $47 + 8$ that would effectively prevent instant recall but open up the possibility of a non-strategic algorithmic method.

Conclusion

The enhanced professional development model using interviewing and video clips has become part of the culture of facilitation in the New Zealand Numeracy Projects - new facilitators are inducted in to its use as a part of their training. Though this varies by region the essentials of teachers viewing video clips, watching facilitators model interviews, and then interviewing students themselves, now are an essential feature of the professional induction of teachers into the Numeracy Projects. This stems from initial evidence that teachers were usually unaware of the presence or the significance of students strategic thinking derived initially from the work of Steffe et al (1983), and that involving teachers with real children, both on video and by interviews, is a powerful first step towards raising this awareness and changing their practice in the teaching of mathematics. Subsequently, as part of the professional development offered for the Projects, a number of video clips of students was produced to show teachers what range of number strategic thinking they might expect in their classes and enable them to reflect upon and improve their teaching practices.

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