REFLECTIONS FROM THE ANALYTIC GEOMETRY COURSES BASED ON CONTEXTUAL TEACHING AND LEARNING THROUGH GEOGEBRA SOFTWARE

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Abstract: Contextual teaching and learning can fill the gap between abstract mathematical concepts and real life practices. Analytic geometry is among the courses which constitutes a gap in this regard. Moreover, when the relevant literature is reviewed, it is seen that researches on analytic geometry mainly focus on achievement and comparing the computer-aided software with other methods. In this context, this study aimed to investigate "How elementary preservice mathematics teachers learn cylindrical and spherical coordination in contextual teaching and learning environments, supported by the GeoGebra software". Case study was conducted. Eight preservice mathematics teachers participated in the study. The instruments of the study were composed of worksheets, semi-structured interviews and models, created through the GeoGebra software. As a result of the study; it was revealed that preservice mathematics teachers can learn cylindrical and spherical coordination in a meaningful and permanent way.
Keywords: Analytic geometry, Contextual teaching and learning, GeoGebra dynamic mathematics software, REACT strategy.

INTRODUCTION

Contextual teaching and learning is the most effective way for students to see the connection between what they learn in class and the real world (Scans, 2000). Glynn and Koballa (2005) define it as making use of different situations which students encounter in everyday life, in terms of transferring course contents. According to Choi and Johnson (2005), contextual teaching and learning denotes using learned concepts and relationships as a means of coping with situations or problems encountered in daily life. Constructing a context involves some difficulties, and it was observed that students can infer what they are given when an appropriate context is constructed (Ingram, 2003; Kuhn & Müller, 2014). Hence, as stated by some researchers, contextual teaching and learning environments are effective in enhancing students' learning (Birchinall, 2013; Ingram, 2003; Pierce, 2013; Satriani, Emilia & Gunawan, 2010). In addition, Cord (1998) emphasizes the importance of contextual teaching and learning that the students did not attend in the learning process of mathematics lessons, they did not have efficient experience by relating, and they were not capable of transferring these experiences to different situations.

REACT is the acronym for five essential student engagement strategies: Relating, Experiencing, Applying, Cooperating and Transferring (Crawford, 2001). Relating, the first step of these strategies, involves a student's attempt to contextualize his preliminary information and acquired experiences (Baltaci, 2014; Crawford, 2001; Costu, 2009; Ozerdem, 2007). In this step, in order to take the student's attention, examples are presented from daily life and lessons are tried to be taught in a selected context (Catlioglu, 2010; Ramsden, 1997). Experiencing is the second step in which students learn and discover knowledge by searching, practicing, and living (Crawford, 2001). In the step of applying, students find a chance to apply the experienced knowledge to the problems by the aid of computer (Crawford, 2001; Ultay & Calık, 2011). In the step of cooperating, students probe in the context of a lesson and share this with other students (Crawford, 2001). In transferring, the last step, students can use their existing knowledge in a new context beside a particular scope (Crawford, 2001; Ramsden, 1997).

Contextual teaching and learning is important in terms of filling the gap between abstract mathematical concepts and real life practices (Cord, 1998). Analytic geometry is among the courses which constitute a gap in this regard. Since most of the analytic geometry concepts are abstract, students have trouble in learning those concepts. Thus, it is crucial to introduce those concepts to students with different methods (Riddle, 1996; Schumann, 2003). Studies show that dynamic software is functional in filling the gap during learning and teaching process of abstract analytical geometry concepts in a traditional paper-and-pencil environment (Baltaci & Yildiz, 2014; Pekdemir, 2004; Saha, Ayubb & Tarmizi, 2010; Schumann, 2003). Therefore this research aims to show that using dynamic software in tandem with REACT strategies will lend an impetus with two powers supporting each other.

Among the present mathematic software, GeoGebra is free, open source-coded, with multiple representation (e.g., different windows for algebraic and geometrical input), and it offers individualized language options, interactive commentary and internet options through which the sources can be shared (Hohenwarter, Hohenwarter, Kreis & Lavicza, 2008). Moreover, GeoGebra software contributes to the modelling of everyday life-related problems (Price & Stacey, 2005). In a similar vein, GeoGebra dynamic mathematics software is a versatile tool for visualizing and objectifying mathematical concepts (Hohenwarter & Jones, 2007).

The writers of the study have observed that the elementary preservice mathematics teachers depended on the formerly learned patterns without questioning. Furthermore, the researchers determined to have difficulties in learning analytic geometry concepts. Similarly, it has been found that preservice teachers have difficulty in constructing because of imagining some expressions (Schumann, 2003). Many researchers have stated that dynamic software contributes in eliminating these kinds of adverse conditions (Frank, 2010; Jahn, 2002). Therefore, the writers wonder about reflections from the analytic geometry courses based on contextual teaching and learning through GeoGebra software.

Literature review shows that studies on GeoGebra software falls under four main headings. The first category is composed of some studies in which a lesson, constructed with GeoGebra, and some variables are examined (Dikovic, 2009; Saha, Ayubb & Tarmizi, 2010). The second consists of the studies in which mathematics teachers and pre-service teachers' views are examined (Aktumen, Horzum, Yildiz & Ceylan, 2011; Kutluca & Zengin, 2011). The third category is composed of some studies in which some variables are examined in the problem solving processes with GeoGebra (Yildiz, Baltaci & Aktumen, 2012). The last one is consists of the studies in which a certain geometric locus is discussed (Antohe, 2009). On the other hand, most of the studies about contextual teaching and learning have been published by the Center for Occupational Research and Development (CORD). By these studies, CORD (1998) aimed to connect abstract mathematics concepts and real-world applications. The study of preservice mathematics teachers is the project of contextual teaching and learning in mathematics, developed at Georgia University. In this project, preservice teachers' beliefs and practices have been investigated in order to emphasize the importance of these beliefs and practices on mathematics education (Glynn & Scott, 2003; Ketter & Arnold, 2003; Tippins, 2003). Moreover, it is seen that learning environments are created with the worksheets formed or the problems based on context in the studies with the REACT strategy (Costu, 2009; Ingram, 2003; Pierce, 2013; Satriani, Emilia & Gunawan, 2010; Yu, Fan & Lin, 2014) and theories and models are put forth with the environment created within the frame of the REACT strategy (Catlıoğlu, 2010). Otherwise, studies about analytic geometry mainly focus on achievement (Erus, 2007; Medrich, Calderon & Hoachlander, 2002) or comparing the use of computer-aided software in teaching analytic geometry concepts with the other methods (Gallou-Dumiel, 1989; Hohenwarter, Hohenwarter, Kreis & Lavicza, 2008; Hoyles & Healy, 1997; Isiksal & Askar, 2005). In this context, this research aims to investigate the reflections in learning process designed in line with contextual teaching and learning and REACT strategies. These reflections were limited to the teaching of cylindrical and spherical coordinates due to the nature of the research. In this context, this study aimed to investigate "*How elementary preservice mathematics teachers learn cylindrical and spherical coordination in contextual teaching and learning environments, supported by the GeoGebra software*".

METHOD

Research Method

The authors of this study have experience on teaching analytical geometry classes, and they published several studies about the analytic geometry teaching. In this research, cylindrical-spherical coordinates were taught in a specially designed learning environment. The lessons carried out by a researcher and case study was used to investigate a certain group deeply and to assess the data without any concern of generalization. According to Yin (2003) a case study design should be considered when: (a) the focus of the study is to answer "how" and "why" questions; (b) it is impossible to manipulate the behavior of those involved in the study.

Participants
Research participants were 8 third-grade preservice mathematics teachers in the 2014-2015 academic years. Four of the participants were female and the other four were male. One of these participants was 20 years old, three of them were 21, and four of them were 22.

When taking courses on basic ICT skills in participants' first year at university, the primary preservice mathematics teachers began interacting with GeoGebra in two courses (General Mathematics, Geometry). Second year, in Calculus I course, GeoGebra is used only for presentation by the researcher when needed such as; examining the derivation of function concepts in table, graph and algebra representation, Rieman integral, the method of rectangle, to draw parametric curve etc. They were guided in these courses by a researcher, who had organized various in-service training sessions on using GeoGebra in mathematics education. Thus, all of the participants were trained in studying with GeoGebra and preparing activities in previous years, they already had the skills needed to construct the situations on the worksheets handed out to them. While deciding on the participants in the research, the researchers took into consideration the academic success levels in analytic geometry, general mathematics and geometry courses and chose students with low, medium and high levels of achievement having the skills of self-expression and being volunteer for semi-structured interview. Besides, all the preservice teachers participating in the research were selected out of those who were good at coordinate system and trigonometry as cylindrical spherical coordinates tried to be taught with a particular learning environment in this research require knowing these subjects basically.

Learning environment was designed for the whole class but, four groups (each group includes two preservice mathematics teachers) focused on data collection. In each group, two preservice teachers worked on one computer. When needed, these groups answered authors' questions and the questions in worksheets using the GeoGebra software. Data were collected about each group during their lessons. In this period, the author took the role of a mentor and conducted semi-structured interviews, in which they recorded the participants' voices by using audio devices.

Data Collection
The research data were collected through worksheets, semi-structured interviews and models formed via GeoGebra software. National and international sources of analytic geometry were used to prepare the worksheets. In direction with these opinions, some changes were done in the guidelines beyond understanding, on the worksheets. In this way, more understandable and clear expressions were used in both mathematics and Turkish.

Three worksheets were used in the application during six course hours. The attainment taken into consideration in these worksheets is as follows: "He can define and interpret that each ordered pair corresponds to a point in R^2 and each ordered triple corresponds to a point in R^3; any point in R^3 is on a right cylinder with center O and the radius r is on the curve of the base of x0y plane; any point in R^3 is on a center O and ρ – radius is on a sphere". In this regard, the preservice teachers were asked to determine the coordinate of a key, which was in a closet and a lamp, which was hung on a wall. By doing this, we aimed to understand preservice teachers' ability to associate objects and their coordinates on the plane and space. Later, the preservice teachers were asked how to determine on the location of an object that was standing on a cylindrical water glass.

Data Analysis
The interviews were carried out with pre-service teachers and the responses were recorded. After each lesson, the participants' worksheets were kept together and they were asked to save and hand in what they brought out on the GeoGebra software. The collected data was analyzed using a content analysis methodology. The content analysis process involves compiling the similar data under certain concepts and themes and to comment on these in a more comprehensive way for readers (Yildirim & Simsek, 2005). Therefore, before the analysis process, the interview data and worksheets for their content were checked. When transcribing the audio data, we presented each conversation without any changes in the order of interviewer and interviewee. Next, the themes were separately developed by the researchers and two field experts. Therefore, the related information was given to field experts, too. To put the analysis into final form, the independent series of analysis were brought together and discussed. At the end of discussions, the determined themes were propounded so as to constitute an answer to the research problem. The reliability of the research was ensured in this way. While presenting the data, each preservice teacher was coded. For example, S1 stands for the first preservice teacher. On the other hand, the indicators of the REACT constituents were created by the help of relevant literature and presented with their codes in the table below.

Table 1: Indicators of the REACT constituents to be used in results.

REACT	Code	Indicators
R	R1	Defining the relationship between previously structured information and those on the GeoGebra screen
	R2	Interrelating the concepts
	R3	Remembering and using previously learned expressions after observations on the software
	R4	Making mathematical comments based on everyday life
E	E1	Experiencing with physical materials
	E2	Comparing estimates and expressions resulting from modelling
	E3	Re-experiencing what has been learnt previously, with the software
	E4	Finding an opportunity to observe the instructions on the worksheets by applying them in the GeoGebra environment
	E5	Changing the points, equations or graphics created on the screen and writing them on the worksheets
A	A1	Doing mathematical generalizations after the experiences on the GeoGebra software as stated in the instructions
	A2	Modelling the given expressions in a paper-and-pencil environment or via the software
	A3	Posing and solving a problem and writing down the algebraic expressions
C	C1	Exchanging ideas and sharing tasks while effectuating the activities
	C2	Checking and comparing what has been done on the worksheets with the feedback on the computer screen
T	T1	Giving examples from everyday life based on the expressions observed on the software
	T2	Reusing a newly learned concept or method in a new expression

FINDINGS

Findings were supported by quotations from the semi-structured interviews, some parts from the worksheets and models formed through the GeoGebra dynamic mathematics software. In the first lesson, we asked the preservice teachers to think about the key closet, made of unit squares and used to open the laboratory doors in the Department (Figure 1), and instructed them to use prepared the guideline. In this guideline, we asked to the preservice teachers how they could determine the coordinate of a key, which was located in the closet. All of the preservice teachers' responses were similar to the following response given by S1:

> *S1: First, I thought the key closet as a matrix. Ultimately, we can define it using columns and rows. For example, the key of this laboratory is located from top to below on the second row and from left to right on the seventh column. Later, my friend and I thought that we may take a starting point and use coordinate axes.*

Thus, they were guided to establish a meaningful relation with the coordinate axis in the plane (R4) with the help of the worksheet delivered. In addition, as seen in the above explanation, the preservice teachers considered matrices in determining the location of the key, and later they established relations with coordinate axes (R2).

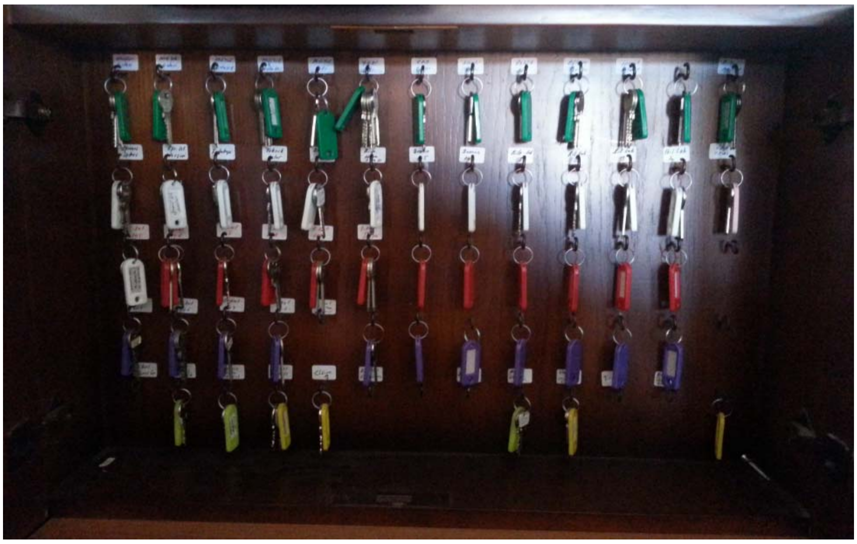


Figure 1: The picture of the key closet located in the department

Next, we asked to the preservice teachers to determine the coordinate of a lamp, hung from the ceiling. While specifying the position of the lamp, most of the preservice teachers said that determining the position of the lamp would be possible by the help of the steps. With stepping, preservice mathematics teachers established a connection from daily life to mathematics in association with coordinate axes (R4). S6 summarize this process as follows:

> *S6: In order to specify the position of a lamp hanging from the ceiling, we think it is possible to reach that lamp by taking eight steps rightward, four steps leftward and, lastly, raising our hand upward. We associated this with the coordinate system. A line along the x axis and raising our hand along the y axis give a line along the z axis.*

At the same time the preservice teachers made various remarks when they were asked about how to address a key on the key closet and a lamp hanging on the ceiling. They called it a different method and said that they were not accustomed to such kind of addressing and relating in coordinate systems (R4). For example, S3 and S5 expressed his thoughts as follows:

> *S3: As a result of the courses taught in this way, I started to think lots of things mathematically as I look at my environment.*

> *S5: In fact, I can say that thinking about the activities was a bit hard and boring because we had not been familiar to such a practice. I completely attribute this situation to the unorthodox and unexpected nature of it.*

Meanwhile, the preservice teachers took this key as a point using the visually of GeoGebra dynamic mathematics software (Figure 2) and thought over how they could address it on both graphics and algebra screen of the software. (R1; R3) For instance, during this process the group formed by S7 and S8 tried to show the location of the key to be on the letter T.

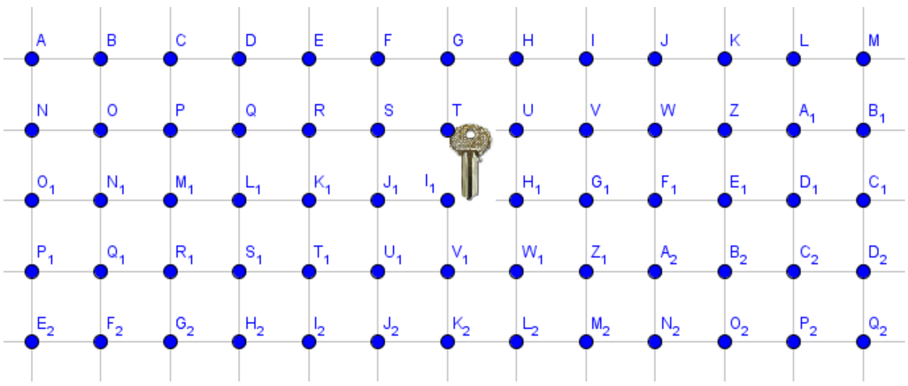


Figure 2: Preservice teachers using the visual quality of the software while addressing a key on the key closet

Thus all of the preservice teachers succeeded in addressing a key in-plane coordinate systems on the GeoGebra screen. Later, we asked to the preservice teachers how they could determine the location of an object, standing on a cylindrical water glass, for a certain time period. From the discussion of the group members, we observed that most of the preservice teachers indicated their inclination to use the height and radius of the glass (R4; E1). For example, S5 and S6's discussion demonstrated this observation.

S5: First of all, we need to use the height because we have to go up in order to reach the object.
S6: Okay. Then, we have to describe the location of the bottom of this height.
S5: What? I did not understand.
S6: I mean, for me, in order to go to the intersection of the height and surface we will need the radius. That is the only way we can describe the location.
S5: Okay.

Next, the preservice mathematics teachers were enabled to study to form cylindrical and spherical coordinates on the GeoGebra software, and to discuss their expressions in group. Having discussed with their group members, the preservice teachers stated that they could address a point on a right cylinder by using its base and height and they could address a point on a sphere using its radius (E1). Reflections from the worksheet of S1 are as in Figure 3.

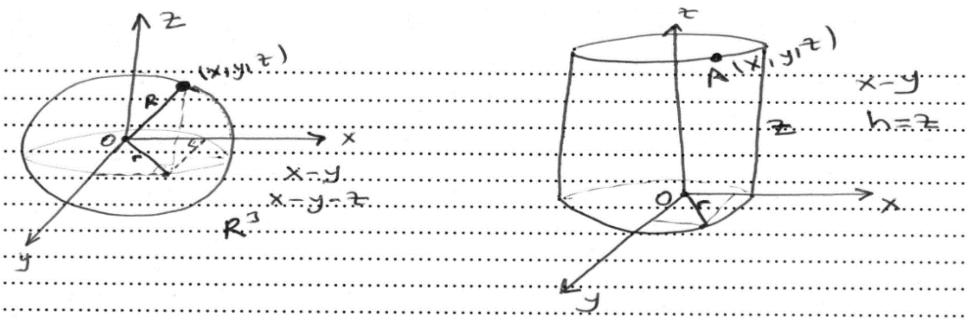
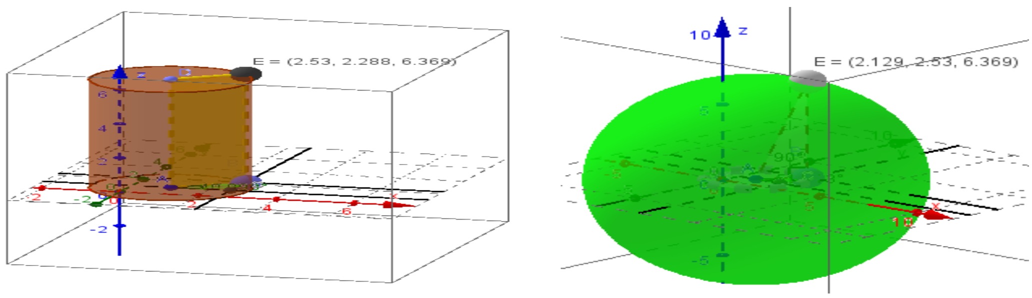


Figure 3: S1's addressing with the forms on the worksheet

After, the preservice teachers were asked to form points on a cylinder and a sphere on the GeoGebra screen. For example, the models formed by the group of S1 and S2 and the group of S7 and S8 are as in Figure 4 (E4; E5).



S1 and S2 trying to experience a point on the cylinder

S7 and S8 trying to experience a point on the sphere

Figure 4: Models formed through GeoGebra

As can be seen in Figure 4, when examined the preservice teachers' figures both on the worksheets and GeoGebra software screen, the preservice teachers were seen to have a kind of experience period by trying to decide the cylindrical and spherical coordinates with using both pen and pencil in addition to computer environment (E1; E4; E5). Besides, S1 stated that the GeoGebra software made it possible for him to visualize his previous reflections on the paper (E3). He summarizes the actualization of this process as follows:

S1: We observed this situation better on the GeoGebra software. Previously we were only envisaging that point but now we are able to make and see it stroll. This made our acts more rational.

In addition, S8 said that they were able to experience their estimates thanks to the dynamic characteristics of GeoGebra software (E2).

> *S8: We had estimated differently before but, at last, we noticed the wrongness of these estimates thanks to the GeoGebra software.*

Thereafter, the preservice mathematics teachers were asked to answer some questions related to the cylindrical and spherical coordinates and they were enabled to apply the expressions they experienced. They could specify the position of a point on a right cylinder by using its base and height values and the position of a point on a sphere by modelling these in a paper-and-pencil environment (A2).

> *S4: We can model the cylinder like this if we assume it a cylinder with a height on the x,y axis, and say it A(x,y,z). We can specify the point on the sphere so as its radius to be R. These denote the points strolling on the sphere and the cylinder.*

Thanks to the feedbacks from the GeoGebra software, preservice teachers had a chance to correct their errors. For instance, while trying to form the cylinder, S7 and S8 mistakenly wrote the values of focal point and height on the GeoGebra input screen and, upon seeing the 'invalid entry' warning on the software. The preservice teachers corrected their mistakes by cooperating in this way (C2; C1).

Having used the GeoGebra software according to the instructions on their worksheets, most of the preservice teachers were able to interrelate and mathematically generalize the cylindrical and spherical coordinates of a point having Euclid coordinates (x,y,z) (A1). Only one group was unsuccessful at this step. For example, the generalization of spherical coordinates by S5 group is shown in Figure 5 and its view on the GeoGebra screen is in Figure 6.

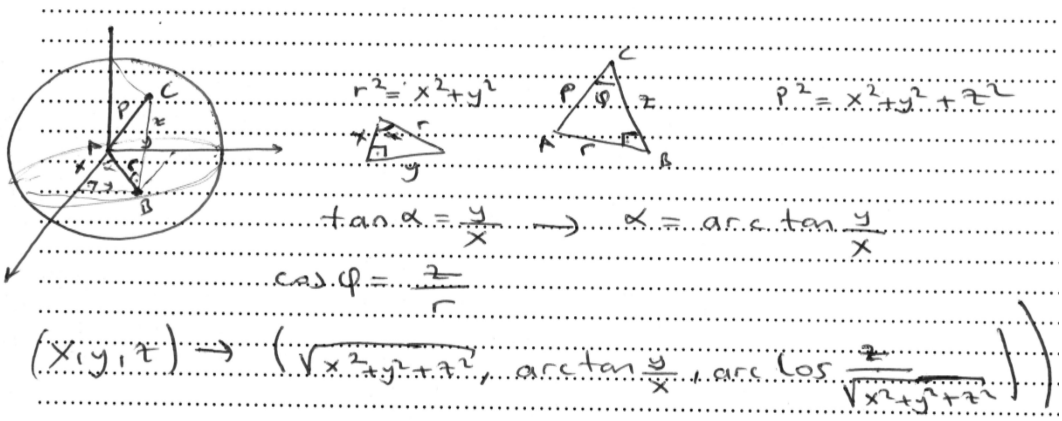


Figure 5: Generalization on spherical coordinates

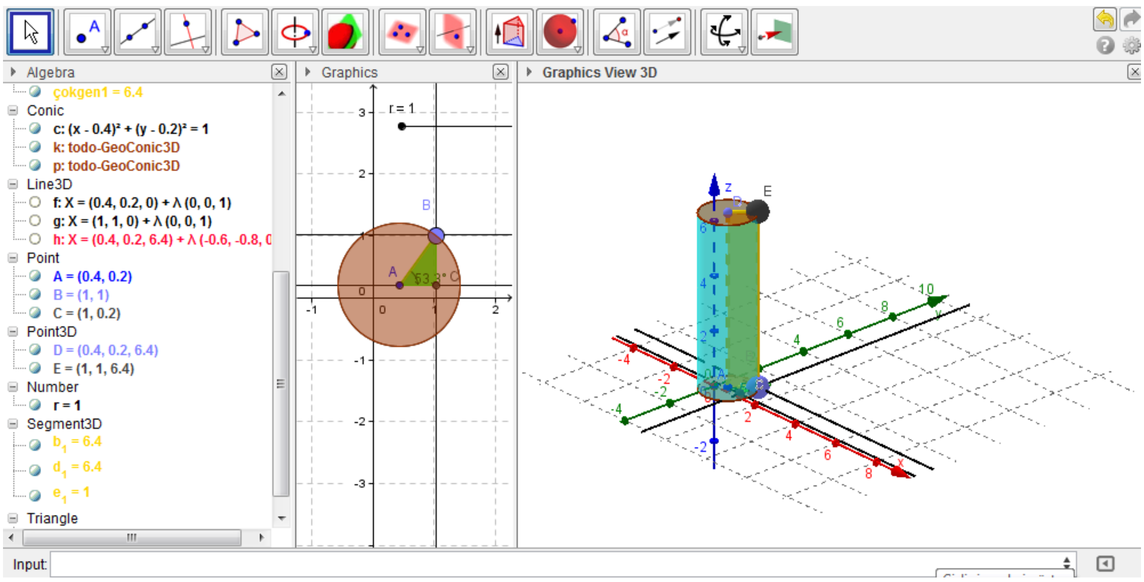


Figure 6: Reflections from the GeoGebra screen in the applying process

Next, the preservice teachers were able to find answers to the posed problems. One of the problems was "*to determine both cylindrical and spherical coordinates of the point A (1,1,3) with your group members*". It is thought-provoking that the problems were formed just by changing the numbers (A3). The answer of S1 group on the problem above is shown in Figure 7 and the model formed by the S1 group is as in Figure 8.

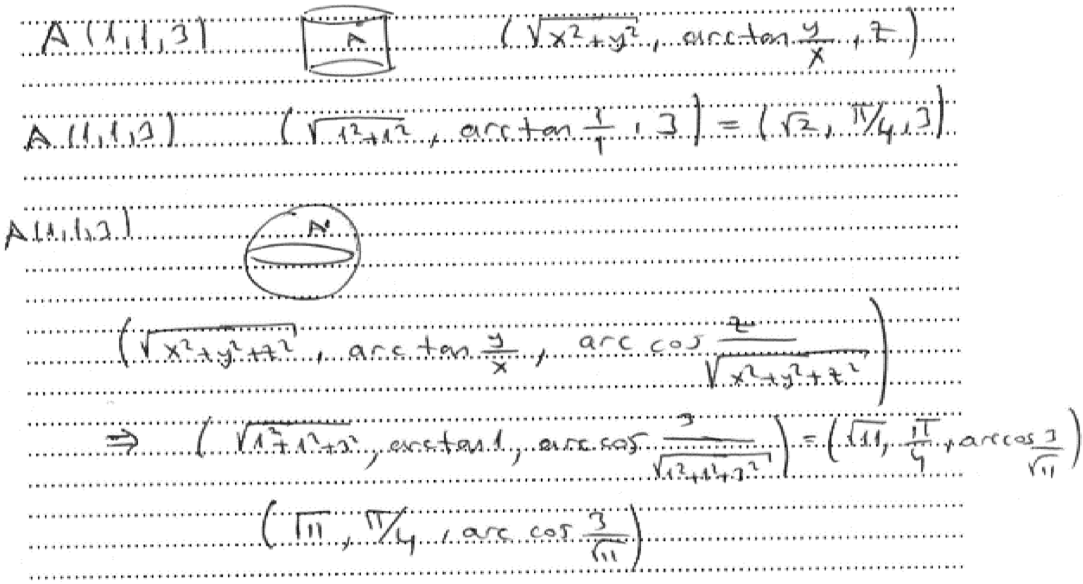


Figure 7: Answer of S1's group about the problem

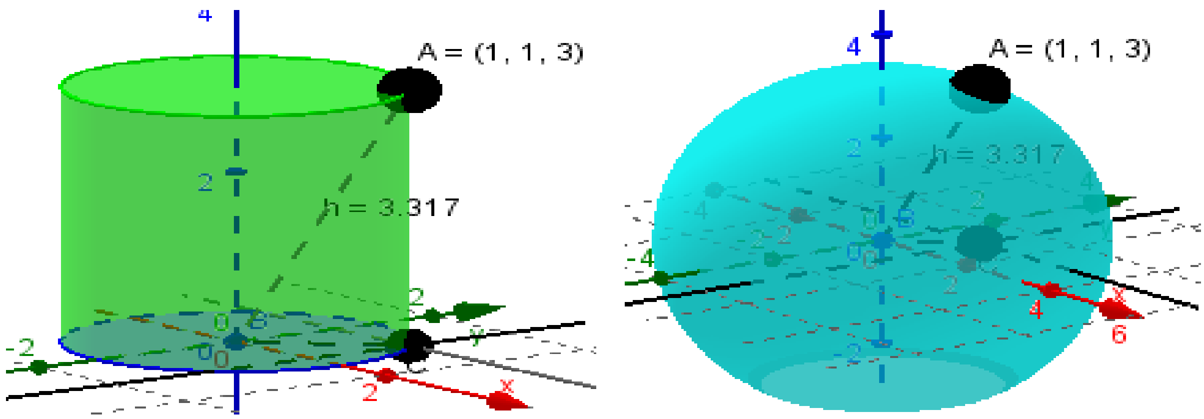


Figure 8: The model formed by S1's group on software in the applying step

In this process, the preservice teachers used the trigonometric expressions they have learnt before to attain the required results by way of right triangles and angles on the GeoGebra screen (R2; R3). S6 express this process as follows:

> *S6: While forming the cylindrical coordinates, we specified the lengths by drawing perpendicular lines to the axes from the point on the circle. I thought of reverse trigonometric expressions using x, y, and arctangent values.*

Thereafter, the preservice teachers noted that any spatial point can be addressed on a right inverse cone in a similar way to the problem above (T2). The figures formed by S3 on both paper and software relating to this result are presented in Figure 9.

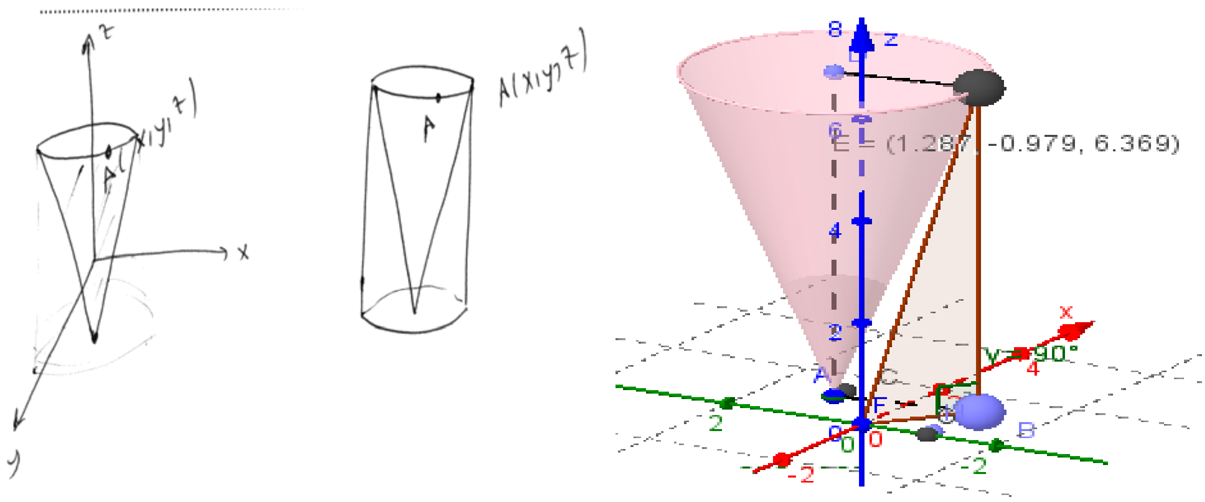


Figure 9: S3 transferring what she knows

In all these activities, the preservice mathematics teachers worked in groups and a well-planned discussion environment was provided so that the cooperating step came to fruition during the course. Also a natural cooperation was provided between the teachers and computer (C1). S7 expressed in the semi-structured interview regarding this finding is as follows:

> *S7: During the lesson, I and my friend tried to do what we are asked. We communicated well but sometimes reasoned differently. However. We checked our conflicting opinions and corrected our mistakes together in the end.*

When looked at the whole semi-structured interview with the preservice mathematics teachers, they said that they can transfer what they learned to other courses and situations (T1). Statement of S3 and S8 is as follows:

S3: It can be applied to geography courses indeed, for example, be used in fixing a place on the globe.
S8:I think it can be used in astronomical research on positions of stars, and weather forecasts as well.

DISCUSSION

It was seen that the preservice mathematics teachers became aware of what they would deal with when they were asked to address a key and a lamp without knowing the subject matter. As supported in a study by Catlioglu (2010), preservice teachers can interrelate to activities even if they are not informed about the subject matter. On the other hand most of the preservice teachers expressed that they enjoyed this atypical method even though they were unfamiliar to this way of teaching. Ramsden (1997) and Tippins (2003) argue in the same way that courses based on contextual learning are found entertaining by students. According to our research results, the preservice mathematics teachers can be adapted to the new method and efficiently accomplish this learning process.

A majority of the preservice mathematics teachers managed to specify the position of a key and a lamp both in plane and in space through coordinate systems using the GeoGebra software. Hennessy (1993), Murphy (1994) and Ayvaci, Ultay and Mert (2013) also conclude that using meaningful and appropriate contexts in learning and teaching may enhance students' ability to relate. On the other hand, most of the preservice teachers tried to determine the location of an object, standing on a cylindrical water glass with the coordinates on space by using height of the glass and the radius of the base of the same glass. During this process, they changed the water level in the glass in order to observe the height, and by doing this, they entered in a situation where they had some experience. In accordance with the findings of this study, Sowell (1989) stated that using concrete materials might help students' developments in mathematics and geometry. Similarly, Butler, Miller, Crehan, Babbit and Pierce (2003) demonstrated that by using concrete materials, students acquired experiences that increased their level of success. Therefore, during instruction, concrete materials should be used for students to have appropriate experiences where instructors have big responsibilities.

The preservice teachers had a chance to experience coordinate systems and the GeoGebra dynamic mathematics software. As a result, they applied the points on both a cylinder and a sphere, and checked their correctness through the GeoGebra software. As pointed out by Anabousy, Daher, Baya'a and Abu-Naja (2014), Gonzàlez and Herbst (2009) and Santos-Trigo and Cristóbal-Escalante (2008), students can discover by experiencing through the GeoGebra software.

Having used the GeoGebra software according to the guidelines on their worksheets, most of the preservice mathematics teachers were able to interrelate and mathematically generalize the cylindrical and spherical coordinates of a point having Euclid coordinates (x,y,z). According to Antohe (2009) and Dikovich (2009), preservice teachers do mathematical generalizations when they follow instructions on worksheets prepared on the GeoGebra software. Thereby, we can enhance preservice mathematics teachers to learn in a meaningful and permanent way when they are given an opportunity to apply on the GeoGebra software in courses with abstract concepts.

In cooperating, the preservice mathematics teachers said they had a chance to discuss the concepts together with both their group and classmates. It is observed that they configured those concepts better in cooperation with a computer so that they corrected some of their mistakes. For instance, one of the groups in this study made a mistake when entering the values of the center point, radius, and height of a cylinder to the GeoGebra and the software showed their mistake. This study and other researches indicate that students make contact with each other when they make a mistake and correct their mistakes in cooperation (Baki, Yildiz & Baltaci, 2012; Rincon, 2009; Saha, Ayubb & Tarmizi, 2010).

All of the preservice mathematics teachers said they can transfer what they have learned to other courses and situations. Some preservice teachers noted that any spatial point can be addressed on a right inverse cone in a similar way to the problem above. However, Hoffman (2003) states that transferring skill is marginal in mathematics as a result of presenting abstract information instead of using an everyday life-related context. But GeoGebra dynamic mathematics software is a versatile tool for visualizing and objectifying mathematical concepts (Hohenwarter & Jones, 2007; Hohenwarter, Prenier & Yi, 2007). Therefore, the courses within the scope of this research can provide the preservice teachers with the opportunity to transfer from daily life to mathematics and from mathematics to daily life interchangeably, so that they are also useful for students in transferring skills.

Based on the results, by revealing the potential of the dynamic geometry software like the GeoGebra, teachers should be given professional education on the active usage of it in the learning process. On the other hand, the role of the GeoGebra software on making contextual connections with the other analytical geometry concepts can be investigated for further research. In addition, researchers can also examine relationships among some variables using the experimental methods.

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