CHARACTERISTICS OF STUDENTS' CRITICAL THINKING IN SOLVING MATHEMATICS PROBLEM

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ABSTRACT
The purpose of this study was to describe critical thinking process of students in solving mathematical problems based on the framework of Polya. Therefore, this study classified as a descriptive qualitative research. These research subjects were 32 junior high school students of grade 7. The data collection begins with students solving mathematical problems about geometry (trapezoid). If in the process of completion, students experiencing the critical thinking they would be selected as the subject of study and further their thinking process explored. The occurrence of critical thinking in students characterized by the student's behavior that constructs and evaluate the strategies used to solve the problem. The results showed that the students' critical thinking process in solving mathematical problems with the framework of Polya include low critical thinking, medium critical thinking, and strong critical thinking.
Keywords: critical thinking, mathematics problem, critical thinking based framework polya

INTRODUCTION

Some researchers have investigated the importance of thinking in mathematics learning. From these results obtained characteristic mistake of students' thinking in constructing mathematical concepts (Subanji & Nusantara, 2013); refractive thinking characteristics of college students in solving mathematical problems (Prayitno, 2016). Thinking process is a mental activity undertaken by college students in problem-solving and can be seen by appeared behavior as the result of task completion (Prayitno, 2015, 2016). Mathematics learning problems in school are complex. Students find many problems associated with the real context. By the time students are given complex mathematical problems, students will sometimes experience confusion so enable the students to get to know the problem. On the other hand, students will identify the necessary materials so that they can find ways/strategies used to handle the problem. Furthermore, students will connect and evaluate the strategy to generate conclusions. Some of the completion strategy produced by the students is a single strategy, a dual strategy and multi-strategy (Prayitno, 2015; Prayitno, Subanji, & Muksar, 2016). Completion strategy created by the students often reflect the way students' thinking. Therefore, students' thinking skills in solving mathematical problems need to be explored.

In mathematics learning, teachers need to emphasize in thinking, communicating or making decisions. If the ability which required is only reading, writing and calculating ability (*calistung*) or emphasis on procedural ability (calculating in solving problems), then our students will be more concerned with the final outcome of the process. They might even ignore reasoning, thinking, communicating and deciding ability. The importance of students to think has been seeded in one of the mathematics learning objectives that are to train how to think and reason in drawing conclusions through research, experiment, showing similarities, differences and consistencies and inconsistencies (Depdiknas, 2006). In addition NCTM (2000) formulate mathematics learning objectives namely: (a) learn to communicate (mathematical communication), (b) learn to reason (mathematical reasoning), (c) learn to solve problem (mathematical problem solving), (d) learn to connect the idea (mathematical connections), and (e) forming positive attitudes towards mathematics (positive attitudes toward mathematics).
The thinking process occurs when a person is given a complex issue, it's likely someone will clash then the person needs to carry out investigations to produce a solution to the problem. According to (Prayitno, 2016) thinking is a mental activity performed by students to solve problems and can be seen from their behavior in form of assignment completion result. In addition, the opinion (Krulick & Rudnick, 1995) states that thinking can be divided into four categories, namely recall, basic, critical thinking and creative thinking. Therefore, this study examines the critical thinking process.

Students' ability in explaining concepts in their own words is evidence that students experiencing critical thinking (Choy & Cheah, 2009). Some experts define critical thinking is a thinking process that aims to make decisions rationally which is directed to make a decision (Ennis, 1996; Facione, 2015; Jenicek, Croskerry, & Hitchcock, 2011; Krulick & Rudnick, 1995; Lai, 2011). In this case, critical thinking is focused in terms of something that leads to a goal. The purpose of critical thinking finally enables to make decisions. In accordance with Choy & Oo (2012) explains that critical thinking is the process of analyzing and making judgments about

what has happened. Pressure on the word "make judgments" were revealed in the definition explicitly states that the decision is part of critical thinking. In addition, Ennis (1996) revealed that critical thinking is thinking rationally or based on reasoning that is focused on deciding what to believe or do. From these definitions, it can be revealed a few things. First, critical thinking is thinking rational (reasonable). Second, critical thinking is also an activity that focused. It shows that a person not only thinks but in thinking about something, someone wants to understand fully. Third, critical thinking is to consider and evaluate information in a way that finally enables one to make a decision. Some decisions must be based on information known to or derived from what is known.

Based on the view of Pagano & Roselle (2009), critical thinking is a process of evaluating the relevant information and opinion collected in the phase of reflection in a systematic, purposeful, efficient way to develop problem-solving skills. The evaluation revealed Pagano & Roselle (2009) is a form of evaluation of various alternative information or settlement obtained from the reflection. In addition, also, Fisher (2001) reveal that critical thinking is characterized by the activity of skilled interpretation and evaluation of the information and statements. The various definitions of critical thinking revealed by experts contain many similarities. if the opinion of several experts (Fisher, 2001; Pagano & Roselle, 2009) equated or compared, it gained critical thinking component, namely the construction and evaluation (Prayitno, 2016), hereinafter, the component is used as an indicator of critical thinking in the study. Construction activity is characterized by constructing solving strategies needed to lead to an answer, while the evaluation characterized by the process of selecting a solving strategy or answers. The selection process is based on a consideration if there is equality of information. Therefore, critical thinking in this study is thinking the process that is characterized by constructing and evaluating the strategy and the most appropriate answer based on considerations (Prayitno, 2015, 2016).

To be able to explore critical thinking, the thing that needed is knowledge on how to portray students' thinking (Prayitno & Suarniati, 2017). A cognitive map is a technique for representing how the subject thinks about a certain problem or situation so that researchers can have a certain attitude for the next step (Ackermann, Eden, Cropper, & Cropper, 2004). Some researchers explore students' thinking by using a cognitive map (cognitive map), including Subanji & Nusantara (2013) using a cognitive map to explore students' mistakes in constructing mathematical concepts. Perdikaris (2011) uses the term cognitive style or thinking style that is used as a mediator to the students' work in solving geometry problems with the theory of Van Hielle, Jacobs, & Schenk (2003) showed that cognitive maps can be used as a guide to get to the next step in order to obtain directions to think further. Moreover, Pena, Sossa, and Gutierrez (2007) revealed the results of studies showed that cognitive map can describe the causality of phenomena and concepts also can be modeled. Therefore, students' critical thinking process in solving mathematical problems can be explored by using a cognitive map.

Critical thinking process can be used to solve mathematical problems. In solving the problem, students can uncover problems solved. In this case, enabling students to understand the problem. Based on this understanding, students devise problem-solving strategies. in strategic planning, students can find the right ideas or solutions that enable the effective strategies to solve the problem. The right idea or solution can be obtained if critical thinking is used in solving the problem (In'am, 2014). Polya (1973) revealed that problem-solving is identified as the ability to (1) understand the problem, (2) plan, (3) implement the plan and (4) checking back. Therefore, the objective of this study is to describe the process of critical thinking of students in solving mathematical problems based on the framework of Polya. The purpose of this study was to describe critical thinking process of students in solving mathematical problems based on the framework of Polya. The established hypotheses were: (1) their differences on the subject of critical thinking, and (2) critical thinking characteristics include strong critical thinking, medium critical thinking, and low critical thinking.

RESEARCH METHODS

Participants

The approach and type of research are qualitative descriptive. This study examines the students thinking the process in solving mathematical problems based on the framework of Polya portrayed through a cognitive map. Subjects were junior high school students of grade 7 of Malang regency.

Instruments

This research instrument, in the form of story problems on geometry (trapezoid) based on the curriculum used in junior high school. The instrument is presented in Figure 1 below.

An isosceles trapezoid-shaped garden with the alignment of the long sides $(x + 4)$m and $(3x + 2)$m. If the distance between the two parallel lines $2x$ m and the area is 180 m^2, What is the perimeter of the garden?

Figure 1. The research instrument

Procedure
Data collection began with giving an instrument to the students to be solved with a time of 20 minutes by raising their voice. This method is known as *think aloud* (Samkof, Lai, & Weber, 2012). After students obtain the solution, researchers verify the completion process of students to obtain answers. If students are experiencing critical thinking (marked by constructive and evaluation activities) in completing the instrument then used as subjects. If there is a mismatch between what is revealed and what is written, the researchers conducted interviews for further investigation. Furthermore, the results of these interviews are used to describe of students' critical thinking process in solving mathematical problems.

Data Analysis
The results of the data collection are the work of students and think aloud interviews and field notes were analyzed based on the stage developed by Creswell (2009).

RESULTS AND DISCUSSION
The number of subjects in this research was 51 students, there are 17 students experience critical thinking is low, 23 students experience critical thinking is medium, and 11 students experience critical thinking is strong. The results of students' critical thinking in solving mathematical problems are presented in Table 1.

Table 1. The Results of Students' Critical Thinking In Solving Mathematical Problems

Critical Thinking		
Low	Medium	Strong
17	23	11
33%	45%	22%

The Analysis Thinking Process of Subject 1 (S1) To Solve Problem
The thought process S1 begins with the, S1 read the questions repeatedly. In this case, the students understand the problem. students understanding shown by the students draw a trapezoid. After describing the trapezoid, students looked at the question again and wrote what about they know on the matter and wrote it by the side of the trapezoid. Students wrote parallel sides $(x+4)$ and $(3x+2)$. While the distance of parallel side $2x$ like as a high and wide trapezoid is 180. In this case, students are able to understand the garden problems by digging information on the matter and to represent it in the form of a trapezoid image.

The next process, S1 began to devise a plan to solve the problem. The first plan created is constructing parallel sides containing variables. This construction is used to indicate which side is unknown. In this case, S1 trying to find any parallel sides and high trapezoid. Furthermore, S1 find the value of x with wide trapezoid $L = \frac{(a+b)}{2}.t$. At the time S1 planned to look for the value of x, through a broad trapezoid there will be construction process value x with the operation of algebra. The construction obtained S1 to factoring. In this case, S1 undergoing construction thinking when planning to determine the value of x.

In the second plan, S1 replaces the variable x with the results obtained. In this case, S1 performed evaluation process to obtain a length trapezoid. S1 thinking back to search perimeter a trapezoid. Perimeter trapezoid is determined by summing the length of the side trapezoid, but there is one side unknown the value. S1 presume that one side of the unknown trapezoid can be found using the Pythagoras concept. This indicates that S1 evaluating several strategies appropriate to look for one side of the trapezoid. preparation of plans by S1 is shown by the results of his think aloud "*Ehh* .. (pause and look at the trapezoid), *there remain the unknown size is oblique side, it could have sought with Pythagoras* (refer to the image)". In this case, S1 undergoing a process of critical thinking when crafting a strategy or plan for a solution the problem.

Under the plan, S1 began to write comprehensive trapezoid $L = \frac{(a+b)}{2}.t$. S1 replace a with $x+4$ and $b$with $3x+2$. In addition, S1 replaces L with 180 and t with $2x$. S1 uses distributive properties to determine the value of *x*. Following the results of S1 using distributive properties.

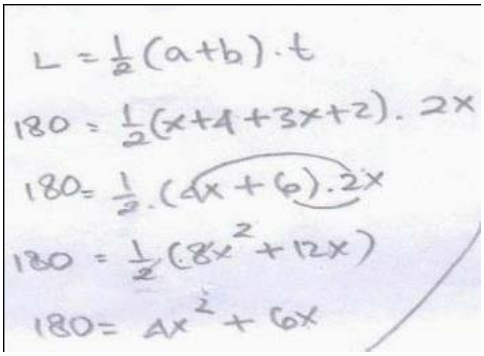

Figure 2. The Work of S1 Using Distributive Properties

The next process, S1 evaluating $4x^2 + 6x = 180$. S1 uses the concept of factoring to determine the value of x. furthermore, S1 simplifying thus obtained $2x^2 + 3x - 90 = 0$. Form of the equation $2x^2 + 3x - 90 = 0$ if factored so $(x + 6)$ and $(2x + 15)$. Of these factors, obtained the value of *x*. Here are the results of the students working to determine *x*.

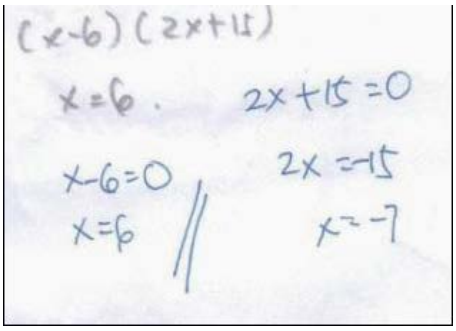

Figure 3. The Work of S1 When Factoring

From the results of the above work, the value of *x* is *6* and *-7*. S1 returned to the evaluation process of the value of *x*. S1 select *6* qualified for the value of *x*, it is due to positive numbers can express the length of a side. The following excerpts of an interview with a researcher S1.

P : *What do you mean about 6*
S1 : *Cause as qualified for the length of the side is a number which is a positive value.*

The next process S1 substitution *x= 6* on parallel sides and high of the trapezoid. By the time students substitution *x = 6* on parallel sides and high of trapezoid occurred construction process in critical thinking S1. S1 continue the process of determining the hypotenuse. The students began to think, "hypotenuse using the Pythagoras theorem". The next S1 uses the concept of Pythagoras when looking for the hypotenuse of a trapezoid.

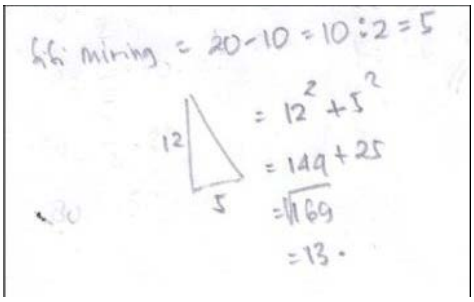

Figure 4. The Work of S1 Using Pythagoras Concept

All sides of the trapezoid are already in to know, so S1 carry out the next plan is to calculate the perimeter of the pool which is trapezoid in shape. S1 see elements that were asked, S1 see the images back and look at the result of the substitution of the sides of the trapezoid to calculate the perimeter of a trapezoid "for all sizes of the sides has been found then calculate its perimeter by summing all sides of the trapezoid". This is the process of construction in the process of critical thinking. The process of the student to check his work by reading back in the matter, then look back at the known and asked, looking back at his work carefully, it is a process of

evaluating the students critical thinking. S1 substitution value of x on the side and high of the trapezoid, then it substitution to the trapezoid area. When the trapezoid area worth *180*, so the resulting answer is true and correct. Therefore, S1 classified into strong critical thinking. Following the results work S1 when checking back.

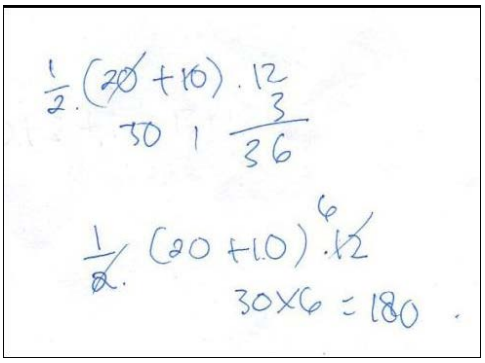

Figure 5. S1 when checking back

The Analysis Thinking Process of Subject 2 (S2) To Solve Problem

S2 began to read about repeatedly. S1 begin to understand the problem. Afterward, S2 can be shown what is known and who asked the question. Here are the results of the work in stages S2 understand the problem.

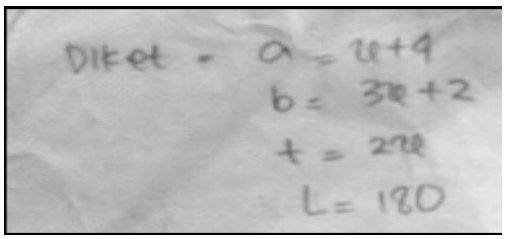

Figure 6. The Work of S2 When Understand the problem

After understanding the given problem, then S2 plan to problem-solving the given problem. The composition of the S2 plan made is to find the value of x through broad the trapezoid$L = \frac{(a+b)}{2}.t$. In this case, S2 used trapezoid area formula for determining the value of x. In this process, S2 make the evaluation process of the strategy that will be used to resolve this problem. The value of *x* is found later on substitute into the parallel sides and high of a trapezoid. This process is known as the process of constructing the value of *x* to get the length and side of the trapezoid. Here, the results of think aloud S2 when doing the planning strategy "*looks like, should have to sought the value of x first. The value of* x *can be obtained by using the formula trapezoid. then substitute to each side of the trapezoid, the next step is determined perimeter the trapezoid of the value of x?*

The results statement from S2, clarifying that the value of x then can be used to determine the perimeter of a trapezoid. In this case, S2 can do the planning of the strategy that will be used to determine the perimeter of a trapezoid. The next process, S2 begin to resolve the matter based on plans that have been prepared. S2 began to write the formula $L = \frac{(a+b)}{2}xt$. S2 operate the wide of trapezoid to determine the value of x. when doing completion, S2 encountered an error in operating the$\frac{(x+4+3x+2).2x}{2}$. S2 indicates that the result $\frac{(x+4+3x+2).2x}{2}$ is $4x + 6x$. however, the completion which is considered S2 one seems to have improved. This, due S2 reflect on the results of his work. The next step, S2 perform factoring towards completion.

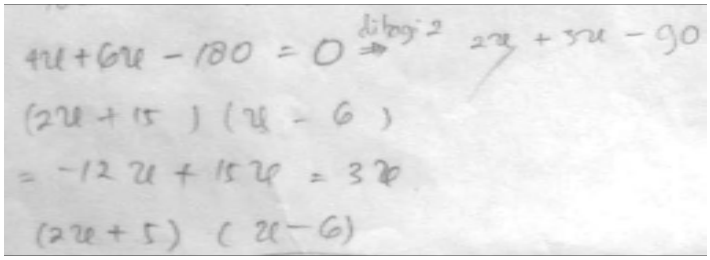

Figure 7. The Work of S2 When Factoring

At the time the construction process occurs factoring in critical thinking, but the criticality of its low because by the time he was factoring $2x + 3x - 90 = 0$ the result is incorrect. while $(2x + 5)(x - 6)$ is a factor of $2x^2 + 3x - 90 = 0$. This shows that the S2, experienced a low critical thinking when solving problems.

P : *Are you confident with the results?*
S1 : *Tough back in the settlement (Oh yes, I forgot to write his rank second sir, so in this equation is no rank. So the result is this factor ".*

The interview results showed that the S2 experienced reflective thinking. In this case, S2 wrong in writing but when the researchers questioned the results of its work, S2 understand the fault location and repair it. The value $x = 6$ were then substituted on the high side and the parallel trapezoid. so that each side acquired *10* and *20*. While high of the trapezoid is *12*. Then S2 continue the next plan is to look for the hypotenuse. The students began to think back and remember the Pythagoras theorem. The next S2 uses the concept of Pythagoras, when looking for the hypotenuse of a trapezoid.

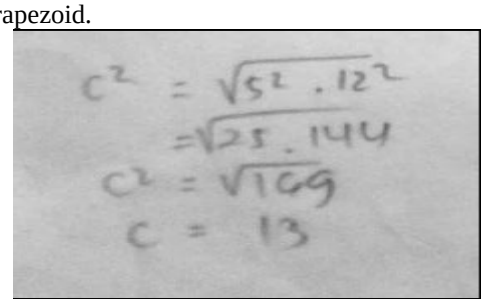

Figure 8. The Work of S2 Using Pythagoras Concept

Based on the hypotenuse, S2 can determine the perimeter of a trapezoid. Roving trapezoid is *10 + 20 + 13 + 13 = 56*. In this case, S2 can solve the problem of the trapezoid (geometry). The next step the S2 is to re-examine his work is read back the matter, then look back at the known and asked, looking back at his work carefully, it is a process of evaluating the student's critical thinking. Therefore, S2 is classified into low critical thinking.
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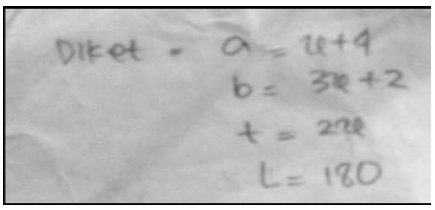

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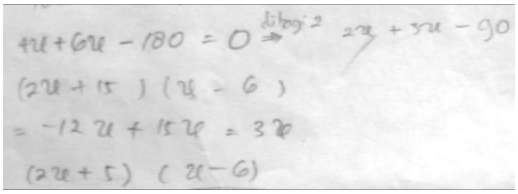

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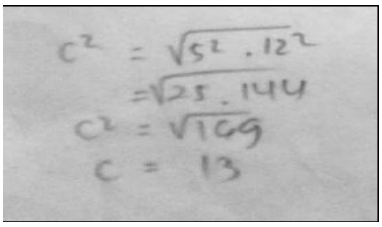

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The Analysis Thinking Process of Subject 3 (S3) To Solve Problem
The process critical thinking of S3 begins with manufacture a trapezoid image and write the parallel sides *(x + 4)* and *(3x + 2)* and the high is *2x*. In this case, S3 represents a garden problem with trapezoid and the elements are known. The next process, S3 settlement plan by first determining the value of x that side of the trapezoid can be known. S2 suspect that one side of the trapezoid can be obtained from the trapezoid area and Pythagoras. This is shown by the results of *think aloud S2 "hmm ..advance determined side and height. It can be searched by trapezoid area and Pythagoras".*

The next process, S3 began to devise a plan to solve the problem. At this stage, S3 identifies that the parallel side alignment of an element x and the long sides are unknown. S3 seem to think that to determine the value of x based on the high and the parallel side of trapezoid required the wide of trapezoid$L = \frac{(a+b)}{2}.t$. When students are planning to look for the value of *x*, with anwide of a trapezoid so the algebra operation process will occur until the factoring of algebraic form. In this case, S1 is able to plan well when looking for value *x*.

Under the plan, S3 began to appear to write $L = \frac{(a+b)}{2}.t$. S1 looks associate with known elements trapezoid area. this continues until the operation process algebra and obtain $4x^2 + 6x = 180$. S3 looks reshaped the equation $4x^2 + 6x = 180$ to $4x^2 + 6x - 180 = 0$. Based on the general pattern $ax^2 + bx + c = 0$. After that, S3 factoring $4x^2 + 6x - 180 = 0$ by considering the concept of factoring and dividing both sides by *2* to obtain $(2x + 15)$ and $(x - 6)$. These factors, the value of *x* is obtained. Here are the results of the students' work in determining *x*.

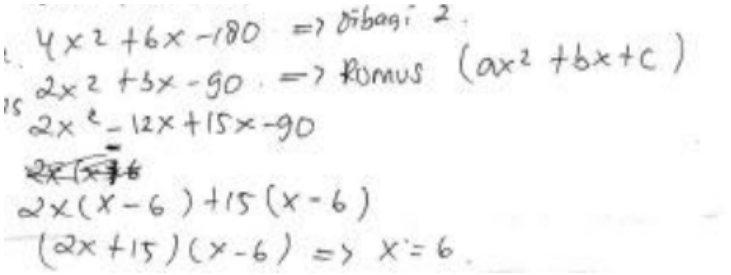

Figure 9. The Work of S3 When Factoring

From above the results work, the value of *x* is *6* and *-7*. From the above results, these students choose the positive value because it is more qualified to determine the value of *x*. The following excerpts of an interview with a researcher S3.

P : *This keeps why 6 that you use as the value of x and not -7.5?*
S1 : *Because as eligible for the length of the side is a number which is positive.*

The next process S3 substitution *x = 6* on parallel sides and tall trapezoid. By the time students substitution *x = 6* on parallel sides and high on trapezoid occurs construction process in critical thinking S1. S1 continue the process of determining the hypotenuse. The students began to think back and remember the Pythagoras theorem, *whether by using the Pythagoras theorem, his side will be found ?*. By the time the students had occurred the evaluation process of the completion strategy. Following the work of S3 by using the Pythagoras concept when looking for the hypotenuse of a trapezoid.

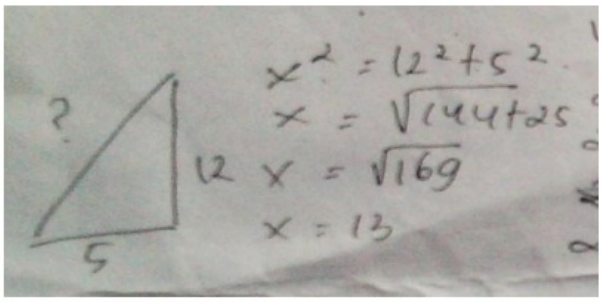

Figure 10. The Work of S3 Using Pythagoras Concept

Since all sides of the trapezoid is already in the know, then S3 executing the next plan is to calculate the perimeter of the pool which is trapezoid in shape. S3 see elements that were asked, S3 see the images back and see the results of the substitution the sides of the trapezoid to calculate the perimeter of the trapezoid *"The next step is for all sizes of the sides has been found then calculate its perimeter by summing all sides of the trapezoid"*. This is the construction process in the process of critical thinking. In determining the perimeter, there are mistakes made by S3 that when summing all sides, but this does not affect the process of completion. Here are the results of the answers S3 when determining the perimeter of a trapezoid.

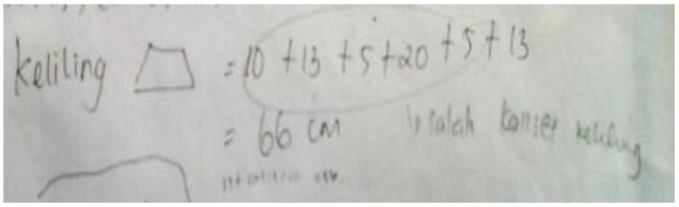

Figure 10. The Work of S3 When Determine Perimeter

The process of the student to check his work by reading back the question, then look back at the known and asked, looking back at his work carefully, it is a process of evaluating the student's critical thinking. S3 substituting the value of *x* on the side and high the trapezoid, then substituting in the trapezoid broad. When the broad of the trapezoid are valuable *180*, then the answer is true and correct. Therefore, S3 is categorized into medium critical thinking. Following the work result of S3 when checking back. From the above analysis, the process of student's critical thinking in solving mathematical problems based framework Polya as follows.

Understanding the Problem: Subjects who understand the problems that S1 and S3. The both subjects can identify elements that are known and asked. The both subjects are also able to interpret the problem in the form of pictures, as well as the two subjects, were able to make a hypothesis or conjecture. While S2 only able to identify the elements that are known and asked. In the process of construction has not been seen on problem-solving.

Make Plan: In making plans to solve the problem, three subjects were able to make the good plan. It is shown that all three subjects read back problem, read back the elements that are known and asked, analyze the problems with the plans to be made and continue the next plan.

Implement Plan: In the process of implementing a plan, S1 was very thorough and the final result obtained is also true. S2 is able to describe the back matter. S2 is less rigorous because of an error in the algebra operations analysis but the end result is obtained correctly. The results obtained properly, and lack of the evaluation of critical thinking because they do not read back the completion of the first plan. S3 less scrupulous when calculating the perimeter so that the final result obtained was wrong.

Checking Back: S1 and S3 were able to prove the truth of the answers that he obtained by identifying the elements that are known. The both of these subjects were able to prove to correct the trapezoid broad in accordance to given problems. S2 was not able to prove the truth of the results obtained.

CONCLUSION

Based on the analysis concludes that the critical thinking process is classified into three, namely low critical thinking, medium and strong. Strong critical thinking is characterized by the formation in the form of the data and the facts are clear, precise, thorough and relevant and resolving problems with the viewpoint of a clear and comprehensive. Low critical thinking is marked with information such as data and facts obtained from about less conscientious and less relevant and able to solve the problem with a clear viewpoint and limited. Medium critical thinking marked with information such as data and facts obtained from the less scrupulous about the final results and was able to resolve the problem with a clear viewpoint and limited.

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