# A COMPARISON OF CURVE INTERPOLATION ALGORITHMS FOR LOW CURVATURE CURVES

Vojtech Wrnata[1], Petr Kretschmer[2]

[1]Technical University of Liberec, Faculty of Mechatronics, Informatics and Interdisciplinary Studies, Studentska 1402/2, 461 17, Liberec, Czech Republic
vojtech.wrnata@tul.cz

[2]Technical University of Liberec, Institute for Nanomaterials, Advanced Technology and Innovation, Studentska 1402/2, 461 17, Liberec, Czech Republic
petr.kretschmer@tul.cz

**Abstract:**This paper presents a comparison of two algorithms for low curvature curves. The two compared algorithms are: linear interpolation and interpolation with Bézier curves. The comparison of the interpolation accuracy is verified on a calculation of the length of the reference curve with different curvature and degree of discretization. Arcs of a circle are used as reference curves. The comparison of the accuracy of the length of an interpolled curve and arc shows that interpolation with Bézier curves is always more accurate regardless the curve curvature.

**Keywords:** Algorithm, Arc length, Bézier, Curves, Interpolation

## Introduction

When modelling nanofiber or microfilament structures an image analysis is used to acquire the geometry of fibre layers. Single fibres are represented by points, which the fibres intersect. These points are then interpolled with curves. An observation of real structures points at the fact that fibres in a structure are laid so that they create curves with low curvature. This paper aims to compare two interpolation algorithms - linear interpolation and interpolation with Bézier curves - in order to find out which algorithm is more accurate for fibre interpolation.
The accuracy of the algorithms is verified on a calculation of the length of reference curves with different curvature and discretization.

### REFERENCE CURVES

### CURVE CURVATURE

Curve curvature C was defined as the ratio of height H and length L, C = H/L, see Fig. 1. The curvature in range 0.2-0.02 was chosen for the curves tested in the article. The range reflects the real material fibres in fibre layers. Fig. 1 shows a curve with curvature 0.2.

  

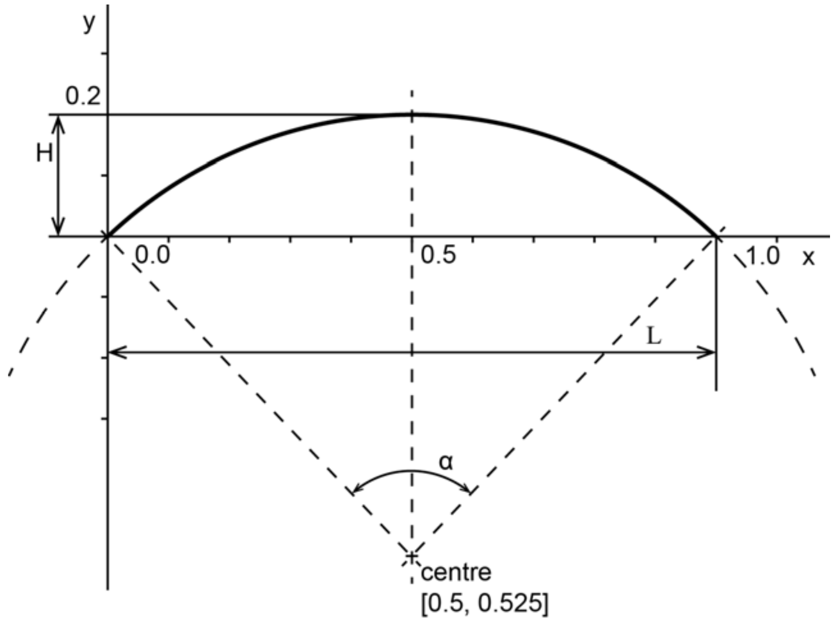


**Fig. 1**. Reference curve with curvature 0.2

**REFERENCE CURVES**

An arc of a circle was chosen as a reference curve. The arc's curvature was changed in the 0.2-0.02 range. The arc of a circle was chosen due to its easy calculation of its length and easy discretization. The discretization step was chosen in the range 5-20. This range reflects real characteristics of interpolled fibres, i.e. it reflects the way how the interpolled points are obtained from real fibre structures.

The arc was chosen so that it always intersects the [0.0, 0.0] and [0.1, 0.0] points. The radius of the circle was calculated so that the centre of the circle was laid on the x coordinate 0.5, y coordinate of the circle centre was calculated for the given curvature of the arc. The length of the curve was calculated from the known coordinate of the circle centre. Table 1 contains data of 10 reference arcs.

**Table 1**. Reference arcs

| Curvature | Arc length | Centre | Angle α |
|---|---|---|---|
| 0.02 | 1.0010663255671826 | [0.5, -6.2399] | 0.15991474849316017 |
| 0.04 | 1.0042612202599455 | [0.5, -3.105] | 0.31931994284894927 |
| 0.06 | 1.009572521275021 | [0.5, -2.0533] | 0.4777157040733537 |
| 0.08 | 1.016980230614833 | [0.5, -1.5225] | 0.6346210487456057 |
| 0.10 | 1.02645691121938 | [0.5, -1.2] | 0.789582239399523 |
| 0.12 | 1.0379682150432712 | [0.5, -0.9816] | 0.9421799228834535 |
| 0.14 | 1.051473519316817 | [0.5, -0.8229] | 1.0920348123468426 |
| 0.16 | 1.0669266439487615 | [0.5, -0.7012] | 1.2388117781698247 |
| 0.18 | 1.0842766217363944 | [0.5, -0.6044] | 1.3822223223268484 |
| 0.20 | 1.103468493625858 | [0.5, -0.525] | 1.5220255084494594 |

**ARC DISCRETIZATION**

The interpolation points are acquired by the arc discretization into a required number of parts. The discretization is conducted by an equidistant partition of the angle α, see Fig. 2. The coordinates of $S_i$ points are calculated as an intersection of the reference arc and circle $x^2 + y^2 = r^2$, where r is the distance from the [0.0, 0.0] point to $S_i$ point, see Fig. 2. The distance $r$ is calculated from the reference arc parameters. The calculation is based on the discretization into 5-20 parts.

  

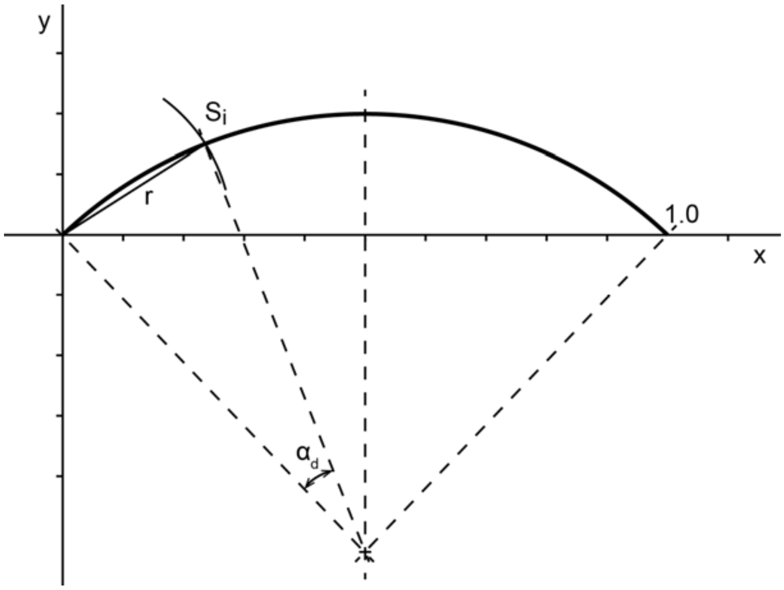


**Fig. 2**. Arc discretization

**INTERPOLATION ALGORITHMS**

**LINEAR INTERPOLATION**

Linear interpolation joints points by abscissae. The length of the arc $L$ is calculated as a sum of the lengths of each part, accordingly to the following pattern:

$$L = \sum_{i=1}^{n} l_i \tag{1}$$

where $n$ is the number of parts and $l_i$ is the length of $i^{th}$ - part.

The length of the $l_i$ part is calculated accordingly:

$$l_i = \sqrt{(Sx_i - Sx_{i-1})^2 + (Sy_i - Sy_{i-1})^2} \tag{2}$$

where $S_x$, $S_y$ are $x$ or $y$ coordinates of the $i^{th}$ - part or $i - 1$ part of the arc.

**INTERPOLATION WITH B-SPLINE CURVES**

Segments of cubic Bézier curves in connection with $C^2$ were used for the interpolation. A segment from a cubic Bézier curve is created between each two points. Its control points are calculated with the help of so-called "A-frame". The algorithm for calculating the control points is described in detail, for example, in chapter 5 of [1] or [2].

The length of the B-spline curve is calculated accordingly to pattern (1), where $l_i$ is the length of an $i^{th}$ segment of the B-spline curve. The length of the segment is calculated with the help of linear interpolation with step 20.

## Results

The results of the algorithms comparison are presented in graphs. The graph in Fig. 3 shows the dependence of relative length deviation ε on the degree of discretization (the number of discretization steps), for the curvature 0.02, 0.1 and 0.2. The graph in Fig. 4 shows the dependence of relative length deviation ε on curve curvature, for discretization 5, 10 and 20. Fig. 5 shows the dependence ration of relative deviations of linear interpolation and B-spline curve interpolation on the degree of discretization (a number of discretization steps), for curve curvature 0.02, 0.1 and 0.2. The ration of relative deviations is defined as:

$$ratio = \frac{\varepsilon_{linear}}{\varepsilon_{bspline}} \tag{3}$$

where $\varepsilon_{linear}$ is a relative length deviation of linear interpolation and $\varepsilon_{bspline}$ relative length deviation of B-spline curve interpolation.

The ration of relative deviations ε is defined accordingly:

$$\varepsilon = \frac{l - L}{L} \tag{4}$$

where $l$ is a calculated length and $L$ is a real length.

  

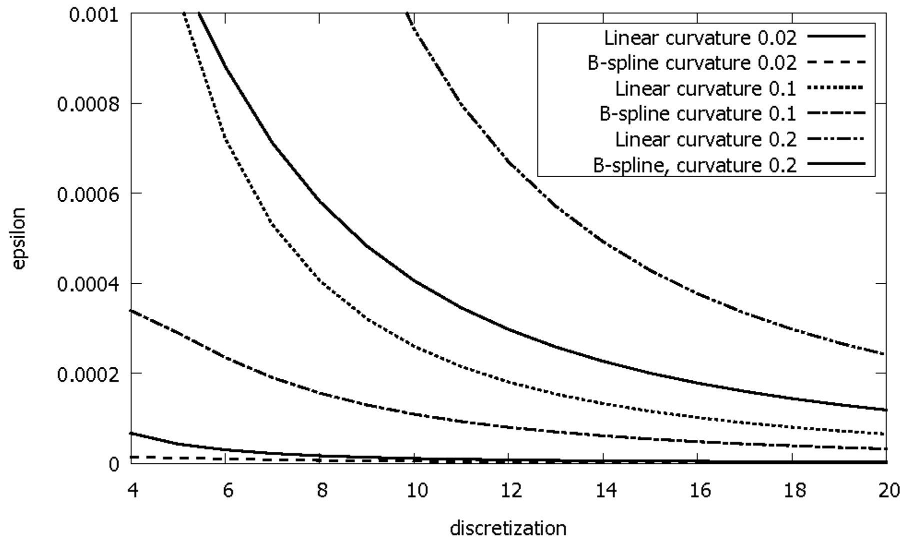


**Fig. 3**. The dependence of the relative length deviation ε on the degree of discretization for curvature 0.02, 0.1 and 0.2.

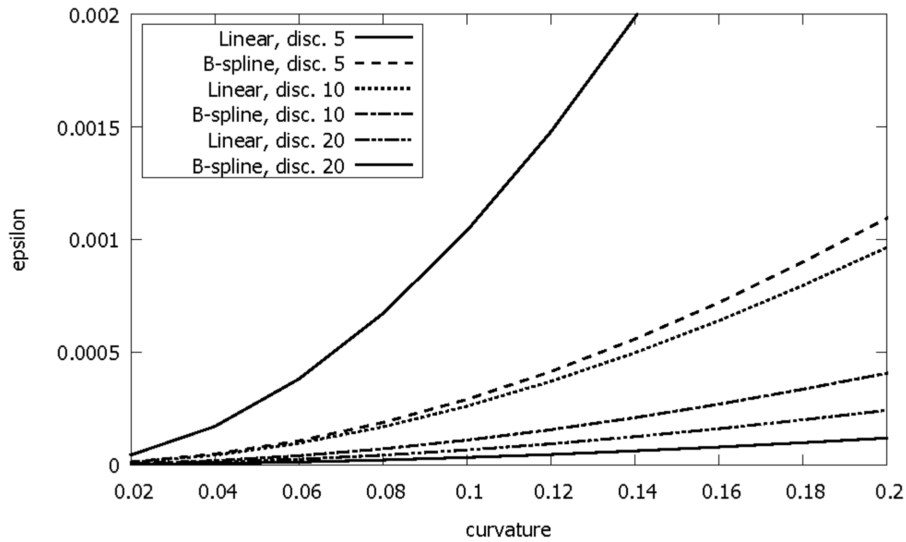


**Fig. 4**. The dependence of relative length deviation ε on the curve curvature for linear interpolation and B-spline curve interpolation.

  

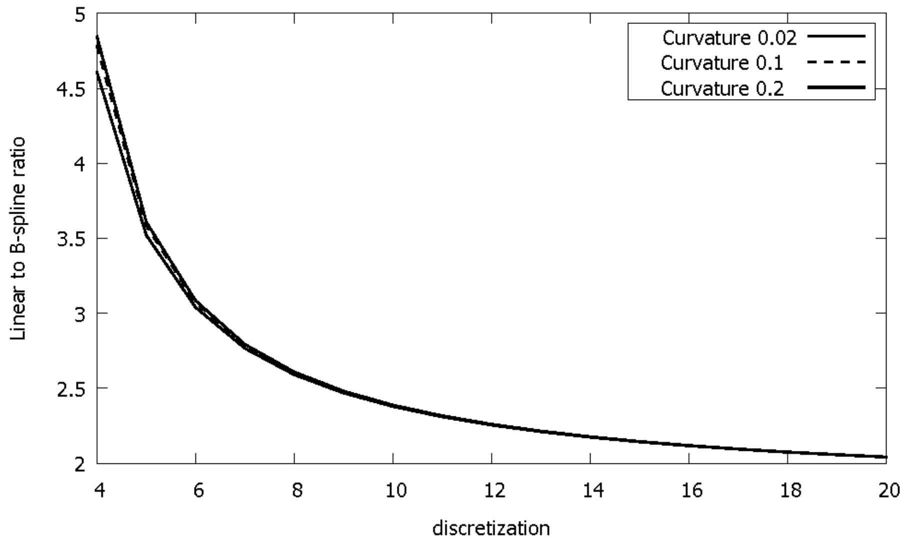


**Fig. 5**. The dependence ratio of relative length deviations of linear interpolation and B-spline curve interpolation on the degree of discretization, for curve curvature 0.02, 0.1 and 0.2.

## Conclusions

The comparison of the algorithms has proved the assumption B-spline curve interpolation is more accurate. The accuracy of the interpolation was verified on ten arcs with curvature from 0.02 to 0.2. The algorithms were verified on all arcs with the discretization range 5-20. The utilization of B-spline curve interpolation was more accurate in all cases.


## Acknowledgements

The work of Vojtech Wrnata was supported by the Ministry of Education of the Czech Republic within the SGS project no. 21066/115 on the Technical University of Liberec.

The work of Petr Kretschmer was obtained through the financial support of the Ministry of Education, Youth and Sports in the framework of the targeted support of the "National Programme for Sustainability I" and the OPR\&DI project Centre for Nanomaterials, Advanced Technologies and Innovation CZ.1.05/2.1.00/01.0005.


## References


Baker, K.A., *The Mathematics of Computer Graphics, Cubic spline curves*, Department of Mathematics, UCLA, Los Angeles, 2002, url = http://www.math.ucla.edu/~baker/149.1.02w/handouts/dd_splines.pdf

Sederberg, T.W., *Computer aided geometric design*, Department of Computer Science, Brigham Young University, 2014, url = http://tom.cs.byu.edu/~557/text/cagd.pdf


  