DETERMINATION OF CONTROL LIMITS FOR ASH CONTENT OF CLEAN COARSE COAL PROCESSED BY HEAVY MEDIUM DRUM

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Abstract: In this research, control limits of the ash content of clean coarse coal product (+18 mm) produced by a heavy medium drum at a coal preparation plant in Turkey was investigated. The importance of data normality and data independence to detect correct control limits of process control chart were shown for ash content of coal product. One year ash data obtained in 2010 which had non-normal distribution and autocorrelated were found to obey lognormal distribution well and ARIMA(1,0,1) model was the best model to remove autocorrelation. Assuming normal distribution and independence, the control limits of ash content were determined as UCL=16.97, CL=12.85, LCL=8.74 with original ash data. When considering only data non-normality and ignoring autocorrelation, the ash control limits were detected as UCL=17.49, CL=12.72, LCL=9.25. On the other hand, the control limits of ash content were implemented as UCL=19.56, CL=12.72, LCL=8.27 if we consider both lognormal distribution and autocorrelation by ARIMA(1,0,1) model. In addition, number of out-of-control points for ARIMA residual chart considering both data non-normality and auto-correlation were less than those obtained by control chart using original data.

Keywords: Non-normality, Autocorrelation, coal preparation, heavy medium drum, ARIMA chart

Introduction

Process control charts (SPCs) are widely used method to monitor and to control of a quality characteristic during an industrial production stage. Control charts provide to monitor the continuous variations in the process and can be applied and interpreted easily (Montgomery, 2011). Its application is based on two basic assumptions. These assumptions are that the data investigated obey normal distribution and independent, i.e not autocorrelated. However, these assumptions should be taken into account and verified prior to generate control charts. How data normality and autocorrelation affect the performance of control charts have been revealed in many scientific papers (Stoumbos and Reynolds, 2000; Castagliola, and Tsung, 2005; Alwan and Roberts, 1988; Bisgaard and Külahçı, 2005; Wheeler, 1991; Srinivasan, 2001; Vermat, 2006; Borror et. al., 1999; Montgomery, 2011; Montgomery, and Runger, 1997; Chou et. al., 1998; Reynolds and Lu, 1997; Lu and Reynolds, 1999; Zhang, 1997; Testik, 2005; Smeti, et. al., 2006, Psarakis and Papaleonida, 2007). It was reported in these works that, if the assumptions are not verified, the control limits determined would not be represent the process correctly and therefore, the resulted SPCs are interpreted wrong by the applicants and incorrect decisions are given about the process. If either of these assumptions is not confirmed, control limits estimated based on original data may not correctly capture the true unusual points. Hence, estimated control limits calculated by verifying assumptions would be incorrect and as a result control charts could be interpreted wrong in terms of their control limits and hence out of control points. To avoid these mistakes, the data should be checked for data normality and autocorrelation.

This research aimed to determine the control limits in terms of ash content for +18 mm clean coarse coal produced by heavy dense drum device. Some examples of SPC charts on different applications of coal production have been carried out by some researchers (Elevli, 2006; Elevli and Behdioğlu, 2006; Deniz and Umucu, 2013; Taşdemir, 2012, 2013 and 2016a). Some studies have also shown that both data normality and autocorrelation affect SPC results seriously in mining and mineral processing applications (Bhattacherjee and Samanta, 2002; Samanta and Bhattacherjee, 2001 and 2004; Elevli et. al., 2009; Taşdemir, 2012, 2013 and 2016a; Taşdemir and Kowalczuk, 2014).

The statistical properties of ash content data used in this research were investigated in detail by Taşdemir (2016b) and determined that the data obey to log normal distribution well instead of normal distribution and also not independent, i.e. autocorrelated. The autocorrelation between consequent ash content data was modelled best by ARIMA(1,0,1) model to achieve data independence (Taşdemir, 2016b). The control limits of SPC under data normality and independence assumptions and also under verification of these assumptions were presented and compared. As a result, correct control limits considering data normality and autocorrelation for the ash content of

+18 mm clean coal produced by heavy medium drum were determined and the correct out of control points were found by ARIMA residual chart.

Materials and Methods

To determine the control limits of ash content for +18 mm coarse clean coals data produced by heavy medium drum, daily data which were obtained in the year of 2010 were supplied by Ege Linyitleri İşletmesi (ELİ) for the Dereköy coal preparation plant in Soma, Turkey. This coal preparation plant has about 4.8 million ton/year coal production capacity. Fig. 1 shows the simplified flowheet of it from Şengül (2008) (Taşdemir, 2016b and 2016c). Rather detailed information about the production stage were given at the first part of this study (Taşdemir, 2016b) and also in (Taşdemir, 2016c). The +18 mm clean coarse coals are floated in the first compartment of the drum and it is shown with a star symbol in Fig. 1. Totally 355 ash content data obtained from the production in 2010 were used in order to determine its control limits.

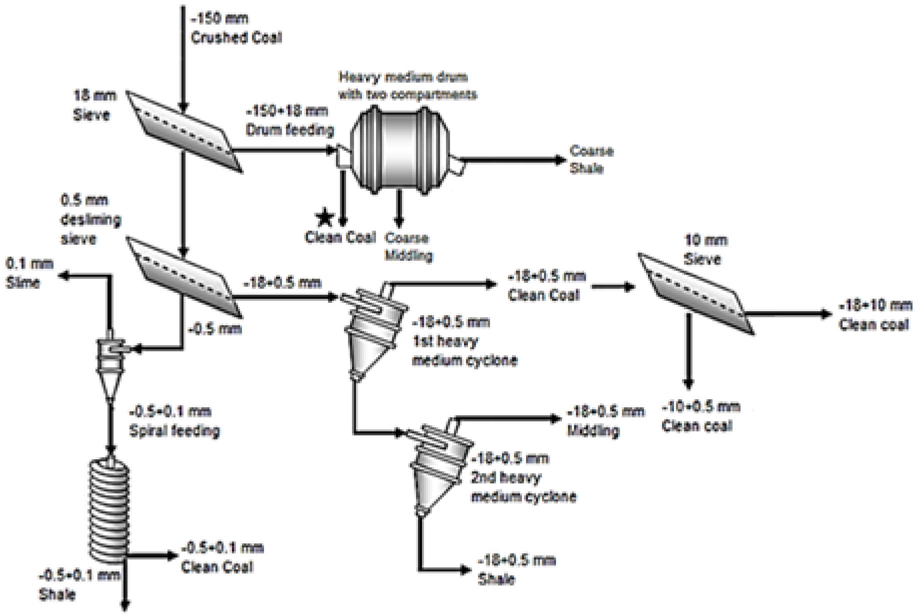


Figure 1. Modified flowsheet of Dereköy coal washing plant from Şengül (2008) and the +18 mm coarse clean coal product of heavy medium drum shown with a star symbol.

During the determination of the control limits of SPC charts, trial versions of Statgraphics XV and Minitab 16.0 softwares were used and SPC charts were generated. The data were found to obey lognormal distribution well to achieve data normality and the ARIMA(1,0,1)time series model was determined the best model based on its lowest AIC (Akaike Information Criterion) to remove autocorrelation (Taşdemir, 2016b). More detailed information for the determination of ARIMA time series models were already documented very well by Box and Jenkins (1976), Montgomery et al., (2008) and Montgomery & Runger (2011).

In this paper, the SPC charts generated under assumptions and verification conditions were presented in order to show the differences of results in terms of control limits and number of out of control points by using the data properties from Taşdemir (2016b).

Results and Discussion

Summary of statistical properties of ash content data

As stated above section, the ash content data have not obeyed normal distribution and lognormal distribution was suitable to make distribution normal (Taşdemir, 2016b). Fig. 2 compares the probability plots of normal and log transformed ash content data with resulted Anderson darling (AD) normality test statistics. The p value of normal distribution is very smaller than 0.05 (<0.005) indicating that the ash content data were not normally distributed. On the other hand, p value of log transformed ash content data distribution is 0.176 ($p>0.05$) showing that the data are represented by log normal distribution well after logarithmic transformation

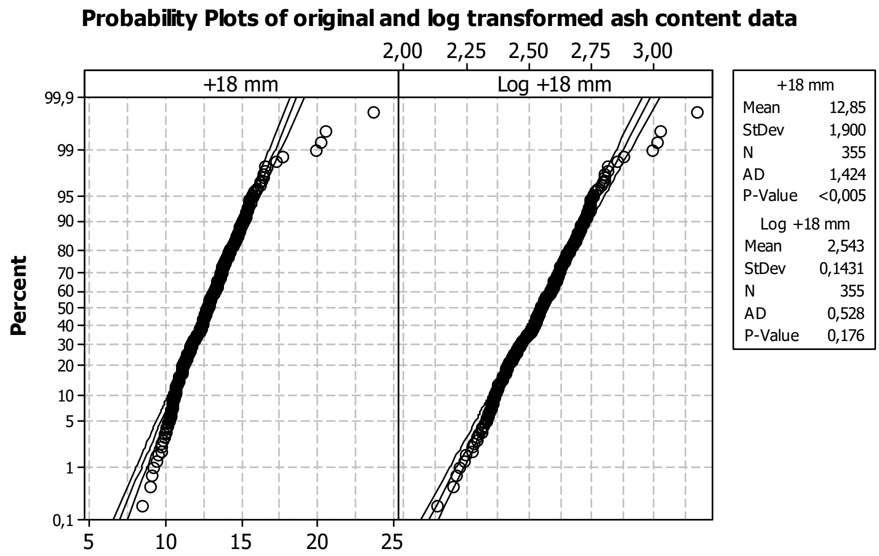


Figure 2. Probability plots of original and log transformed ash content data

Parameters of ARIMA time series model for log-transformed ash content data

After achieving data normality by log transformation, The log transformed ash content of +18 mm clean coal produced by heavy medium drum was found to be modelled by ARIMA(1,0,1) or ARMA(1,1) model well (Taşdemir, 2016b) and the details of model determination can be found there. Table 1 summarizes the ARIMA(1,0,1) model parameters.

Table 1: ARIMA(1,0,1) model summary for log-transformed ash content data (Taşdemir, 2016b)

Parameters	*Estimate*	*Stnd. Error*	*t*	*p value*
AR(1), ϕ	0.7688	0.074445	10.3271	0.000000
MA(1), θ	0.4609	0.102183	4.51042	0.000009
Mean, μ_0	2.5428	0.015762	161.324	0.000000
Constant, δ	0.5879			
WNV*, σ_a^2	0.0166	*: white noise variance		

The ARIMA (1,0,1) time series model is modelled by Eq. 1 as given the following (Castagliola and Tsung, 2005):

$$X_t = (1-\phi)\mu_0 + \phi X_{t-1} + \theta a_{t-1} + a_t \qquad (1)$$

Where X_t is the observation at time t=1, 2, ,…, a_t is the random noise or white noise at time =1, 2, ,… which is assumed to have mean of zero (0) and standard deviation of σ_a, ϕ is the autoregressive parameter of the model which corresponds to p term in the model, θ is moving average parameter which corresponds to q term in the model and μ_0 is the nominal mean of the process (Castagliola and Tsung, 2005). The constant, δ, parameter in the model was calculated from $(1-\phi)\mu_0$.

By using these parameters, following ARIMA(1,0,1) time series model determined for the log transformed ash content of +18 mm clean coal produced by heavy medium drum is given in following Eq. 2 (Taşdemir, 2016b):

$$X_t = 0.5879 + 0.7688X_{t-1} + 0.4609a_{t-1} + a_t \qquad (2)$$

Where, X_t is the log transformed ash content value at time, a_t is the random noise which have distribution of N(0, 0.1289).

Control limits under data assumptions

Table 2 summarizes the individual chart (I-chart) parameters obtained for the original ash content of +18 mm coarse coal under data assumptions, i.e. without data transformation and independence verification. Process sigma (σ) in Table 2 was estimated from average moving range ($\overline{MR}$) for the sample size of 2. The generated SPC and control limits under these assumptions is presented in Fig. 3. According to the Fig. 3, upper control limit (UCL) and lower control limit (LCL) were determined as 16.97 and 8.74 respectively while centre line (CL) or (CTR) was 12.85 which corresponded the mean of ash content data. Six points are out of control limits from UCL and one point is below the LCL.

If we assume that the ash content data had normal distribution and not autocorrelated, the observations of 31, 32, 53, 214, 347 and 348 were beyond the *+3σ* while observation 208 was below the *-3σ* (Fig. 3).

Table 2: I-chart parameters of original ash content data of +18 mm clean coarse product by heavy medium drum

I-Chart Parameters	*Values*
$UCL{=}{+}3\sigma$	16.97
$CL{=}\ \bar{X}$	12.85
$LCL{=}{-}3\sigma$	8.74
$\sigma(\overline{MR}/1.128)$	1.37
$\overline{MR}$	1.55

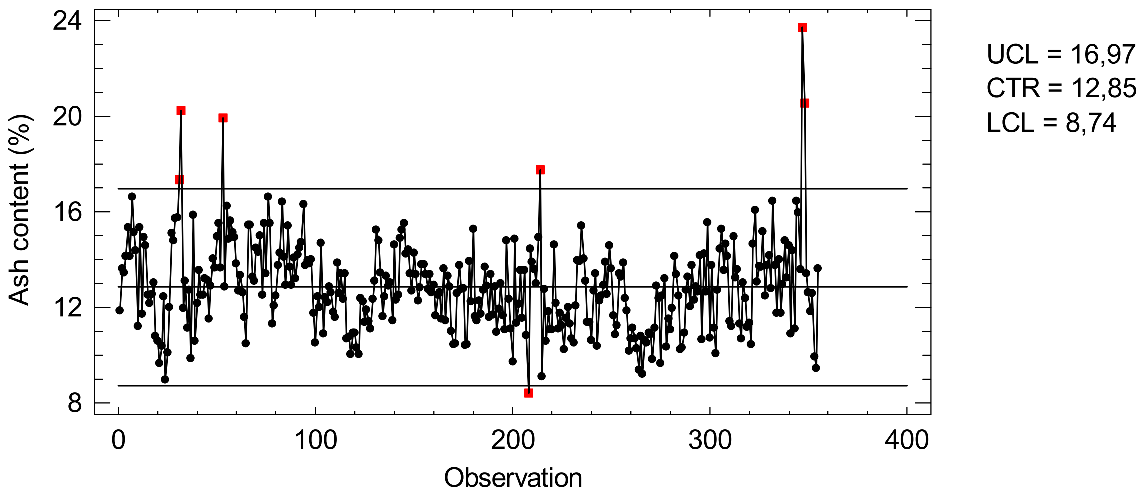


Figure 3. I-chart under normality and autocorrelation assumptions and its control limits

Control limits under verification of data normality and autocorrelation assumption

I-chart parameters after verifying the data normality by log transformation are presented in Table 2 with the corresponding original and transformed metric values. As presented in Fig. 2, the mean of log transformed ash content corresponded to 2.543 in transformed metric. Therefore, back transformed mean of ash content was equal to 12.72 in original metric.

Fig. 4 presents the control limits of ash content on SPC chart generated after data normality as in transformed metric in Fig. 4a and its back transformed metric, i.e. original unit in Fig. 4b. After log transformation, upper control limit (UCL), center line (CL) or (CTR) and lower control limit (LCL) are found as 2.86, 2.54 and 2.22 in logarithmic scale respectively. Since the log transformed values may not be meaningful or not be preferred, back transformed control limits are shown in Fig. 4b which shows the same out of control points with Fig. 4a. As seen in Fig. 4b, UCL and LCL are determined as 17.49 and 9.25 respectively. Compared to Fig. 3, the control limits are very different when the data normality is taken into account and data independence is just assumed. Larger control limits are obtained when data normality is achieved. Total number of out of control limits which beyond control limits are nine, five of them are beyond the UCL and four of them are below the LCL. These unusual points in Fig. 4a and 4b are different from the ones which are obtained in Fig. 3. The observations of 32, 53, 214, 347 and 348 are beyond *+3σ* while observations of 24, 208, 215 and 266 are below *-3σ*.

Table 2: I-chart parameters of original ash content data of +18 mm clean coarse product by heavy medium drum

Parameters	*Transformed metric*	*Original metric*
$UCL = +3\sigma$	2.86	17.49
$CL = \bar{X}$	2.54	12.72
$LCL = -3\sigma$	2.22	9.25

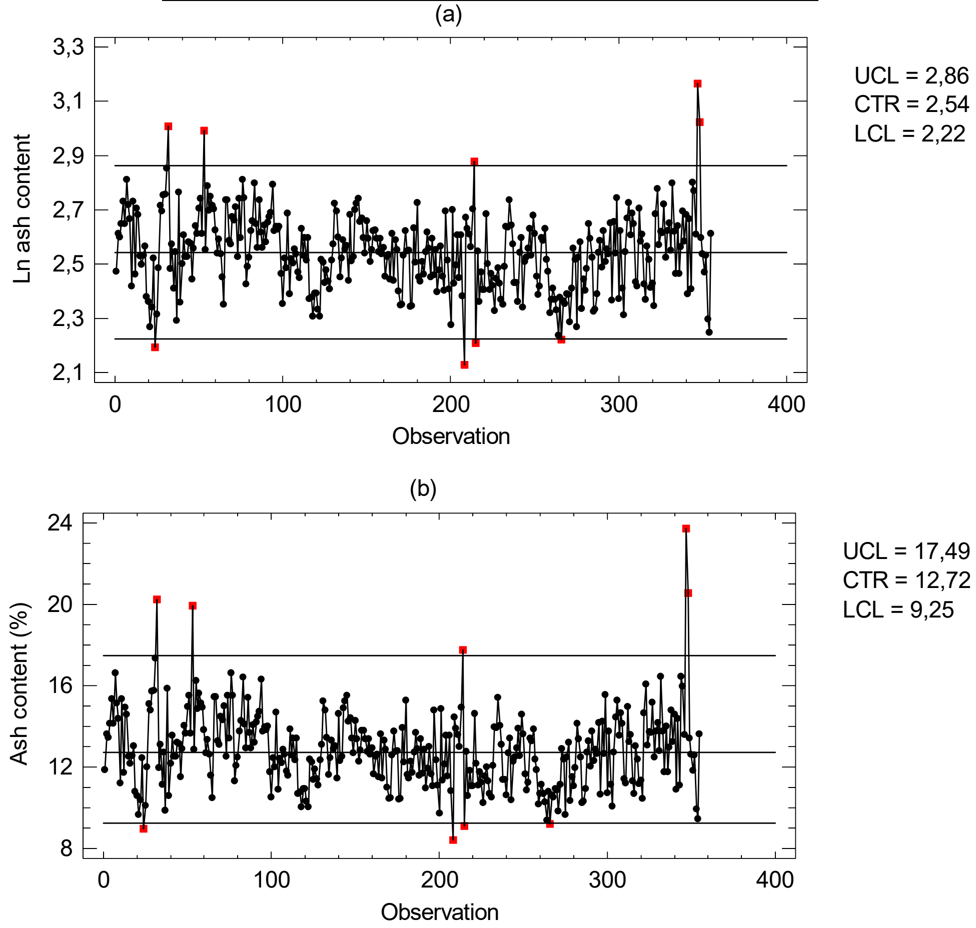


Figure 4. I-chart under normality verification and autocorrelation assumptions in transformed metric (a) and its back-transformed values of control limits in original metric (b)

Control limits under verification of both data normality and autocorrrelation

As stated above, data autocorrelation of log transformed ash content was removed successfully by ARIMA(1,0,1) time series model. While implementing control limits of SPC chart while considering autocorrelation, the process sigma, σ_x, was estimated from the both white noise or random shock and fitted ARIMA(1,0,1) model parameters (Table 1) for +18 mm clean coal product of heavy dense drum. In this chart, center line (CL) was estimated from the following formula;

$$CL = \mu_0 = \frac{\delta}{1-\phi} \quad (3)$$

The CL in Eq. 3 was determined from the parameters of ARIMA(1,0,1) model given in Table 1. Then, the control limits of data, UCL and LCL are drawn around centerline (CL) located at μ_0 by using the process sigma, σ_x;

$$\mu_0 \pm 3\sigma_x \quad (4)$$

The relation between the variance, σ_x^2, of the ARIMA(1,0,1) process, X_t, and the variation, σ_a^2, of the random noise, a_t, is calculated from the following Eq. 5 (Castagliola and Tsung, 2005):

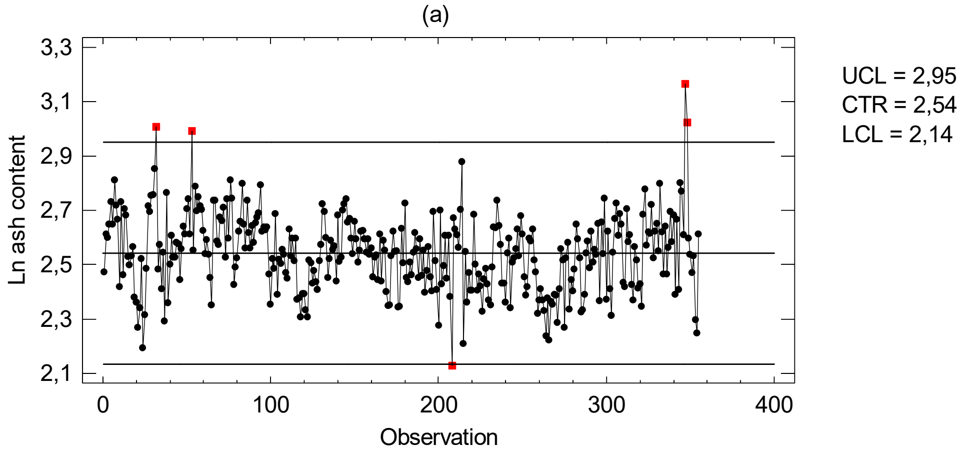
$$\sigma_x^2 = \frac{1+\theta^2-2\phi\theta}{1-\phi^2}\sigma_a^2 \quad (5)$$

The variance of random shocks, σ_a^2, i.e. white noise variance in Eq. 5 can be estimated by both the mean squared error (*MSE*) of fitted AR(2) model and the mean range ($\overline{MR}$) of residuals (Polhemus, 2005). After solving Eq. 5, the variance of log-transformed ash content was about 1.23 times larger than the residual white noise variance ($\sigma_x^2 = 1.2318\sigma_a^2$).

The ARIMA(1,0,1) chart parameters where the random noise variance (σ_a^2) is estimated from the average moving range of ARIMA(1,0,1) residuals ($\overline{MR}$) and then process sigma (σ_x) was calculated by Eq. 5 was given in Table 3. The generated control charts are presented for log transformed metric in Fig. 5a and for back transformed metric in Fig. 5b.

Table 3: ARIMA chart parameters when the white noise variance, σ_a^2 was calculated by average moving range of ARIMA(1,0,1) residuals ($\overline{MR}$))

Parameters	*Transformed metric*	*Original metric*
UCL=$+3\sigma_x$	2.95	19.14
CL= μ_0	2.54	12.72
LCL=$-3\sigma_x$	2.14	8.45



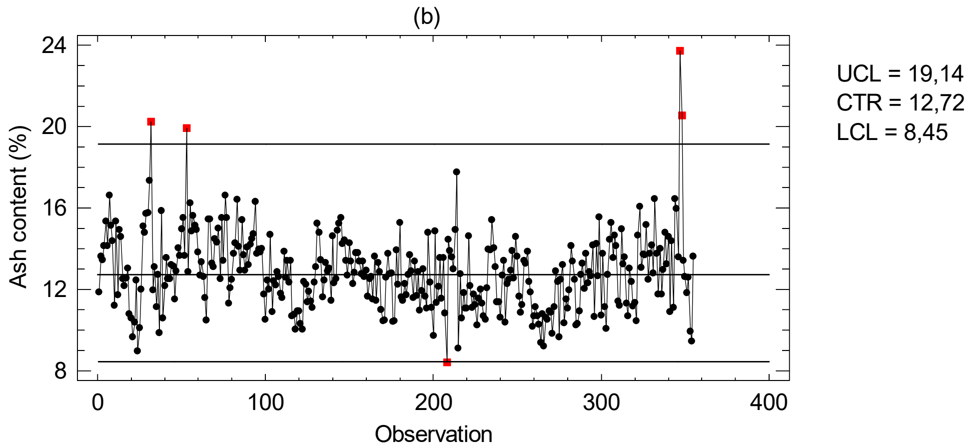


Figure 5. I-chart under normality and autocorrelation verification in transformed metric (a) and its back-transformed values of control limits in original metric (b) (white noise variance, σ_a^2 was calculated by average moving range of ARIMA(1,0,1) residuals ($\overline{MR}$))

Fig. 5a and 5b show the control limits of ash content where both normality and autocorrelation are verified. Compared to Fig 5 with Fig. 3 and Fig. 4, both control limits and out of control points are very different since the data normality and autocorrelation are taken into account when determining the control limits of ash content. The UCL and LCL were determined as 19.14 and 8.45 respectively when the data normality and autocorrelation are verified. The observations of 32, 53, 347 and 348 are beyond the $+3\sigma_x$ while the observation 208 was below the $-3\sigma_x$.

Table 4 shows the control limits of ash content determined for the ARIMA chart parameters when where white noise variance, σ_a^2 was calculated by the mean squared error, *MSE* to determine the process sigma, σ_x. The control limits of generated chart are considered as long term monitoring of the process (Polhemus, 2005). Polhemus (2005) indicates that this control limits are used to determine the process deviations from long-term mean more than expected given the dynamics of the process.

The control limits of ARIMA chart constructed in Fig. 6 shows the long term control limits for the +18 mm clean coarse coal by heavy dense drum. In this chart, process sigma, σ_x was estimated by Eq. 5 and white noise variance, σ_a^2 was calculated by *MSE* of ARIMA(1,0,1) model which was determined as σ_a = 0.1289 and given in Table 1 (Taşdemir, 2016b). As seen from the charts in Fig. 6a and 6b are considerably wider bounds than the charts in Fig. 5 and Fig. 4 since the estimated process σ_x is a function of both white noise or random shock and fitted ARIMA(1,0,1) model parameters (Table 1).

The UCL and LCL were determined as 19.56 and 8.27 for long term control limits. Total number of unusual points was four which are all beyond the UCL which corresponds to observations of 32, 53, 347 and 348.

Table 4: ARIMA chart parameters when white noise variance, σ_a^2 was calculated by *MSE* of ARIMA(1,0,1)

Parameters	*Transformed metric*	*Original metric*
$UCL=+3\sigma_x$	2.97	19.56
$CL=\ \mu_0$	2.54	12.72
$LCL=-3\sigma_x$	2.11	8.27

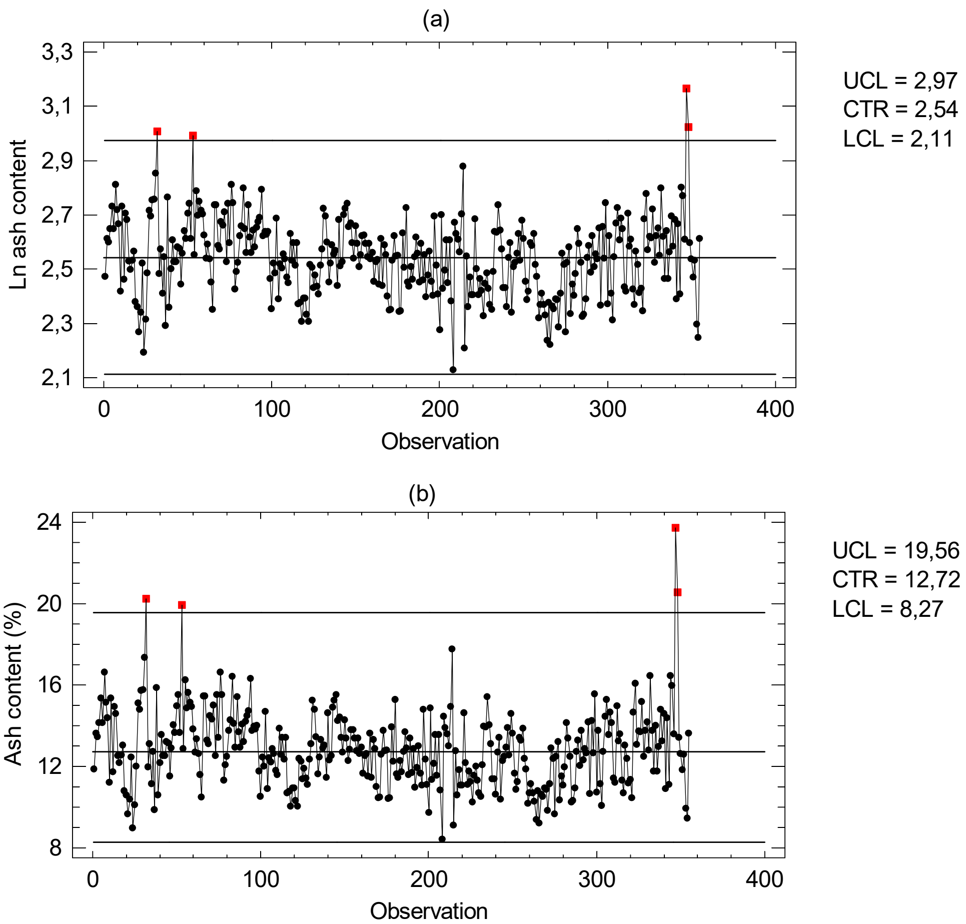


Figure 6. I-chart under normality and autocorrelation verification in transformed metric (a) and its back-transformed values of control limits in original metric (b) (white noise variance, σ_a^2 was calculated by *MSE* of ARIMA(1,0,1))

ARIMA residual chart of log-transformed ash content data

The ARIMA residuals charts were generated to detect real uncontrolled points of ash data. The ARIMA residuals were obtained from the difference between actual log transformed ash values and their forecasted values determined by ARIMA(1,0,1) model. The residuals, a_t, were calculated by rewriting the Eq. 2 as the following:

$$a_t = X_t - 0.5879 - 0.7688X_{t-1} - 0.4609a_{t-1} \quad (6)$$

Since a_t was determined to be independent and identically distributed (*i.i.d*) normal (0, σ_a), control limits of residuals, i.e., UCL and LCL which are drawn around centerline (CL) of zero (0) were calculated by the following Eq. 7 (Castagliola and Tsung, 2005):

$$CL = 0 \pm 3\sigma_a \quad (7)$$

The ARIMA residual chart, where σ_a is estimated from the residual mean of moving range ($\overline{MR}$), resulted for ash content is given in Fig. 7. From Fig. 7, number of points beyond $\pm 3\sigma_a$ limits are 3 which corresponds to observations of 53, 215 and 347. Since the aim is to reduce the ash content as soon as possible during the coal washing process, the out of point beyond LCL (observation 215) cannot be considered uncontrolled process point actually. Therefore, the process can be considered out of control for the 53rd and 347th days in terms of ash content and was in control for the rest days in 2010 based on the ARIMA residuals chart of Fig. 7.

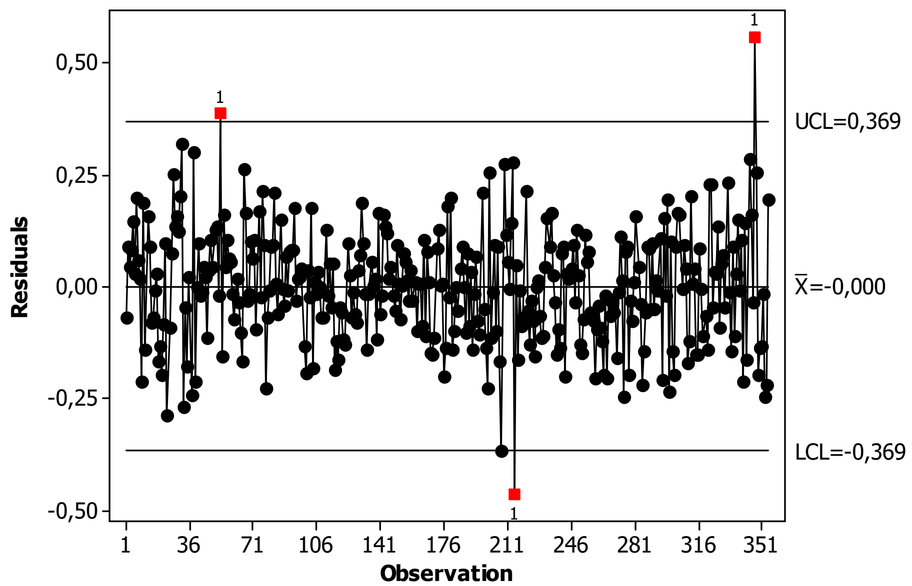


Figure 7. ARIMA residuals chart of ash content data

Conclusions

Control limits ıf ash content for the +18 mm clean coarse coal product produced by heavy medium drum was determined under data normality and data verification conditions. It was shown that the number of total out of control points were different if the assumptions were taken into account or not. In order to give right decision in monitoring and control of a process variable in terms of ash content, both data normality and autocorrelation should be verified prior to application of SPC charts. These can be done for the +18 mm clean coarse coal data by applying log transformation first to achieve data normality and then removing autocorrelation by ARIMA(1,0,1) time series model. Long term control limits were determined as 19.56 for the upper control limit and 8.27 for the lower control limit while center line was 12.72. Only two points were out of control which were determined by the ARIMA residuals chart in 2010 in terms of ash content. This result was different from the ones which were calculated by only assuming data normality and autocorrelation without verifying them.

Acknowledgement

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