ASYMPTOTICALLY $\alpha_f^p(I)$-LACUNARY EQUIVALENT SEQUENCES SPACES

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Abstract: In this study, we define $\alpha_f^p(I)$- lacunary equivalence with order α, and asymptotically α_f (I)-lacunary statistical equivalence with order α,which is a natural combination of the defintion for asymptotically equivalent, Ideal convergence, Statistically limit, Lacunary sequence, Modulus function and a sequence of positive real numbers p=(p_k) and give some relations about these concepts.

Keywords: Asymptotically equivalence, Ideal convergence, Lacunary sequence, Modulus function, Statistically limit.

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Introduction

Let s, ℓ_∞,c denote the spaces of all real sequences, bounded, and convergent sequences,respectively. Any subspace of s is called a sequence space. A lacunary sequence is an increasing sequence θ=(k_r) such that k_0=0, $h_r = k_r - k_{r-1} \rightarrow \infty$ as r→∞ .The intervals determined by θ will be denoted by $I_r=(k_{r-1},k_r]$ and $q_r = k_r/\ k_{r-1}$ .These notations will be used troughout the paper. The sequence space of lacunary strongly convergent sequences N_θ was defined by Freedman et al.(1978), as follows:

$$N_\theta = \{x=(x_i)\in s:\ lim_r h_r \sum_{i\in I_r}|x_i - L|=0 \text{ for some } L\}.$$

The notion of modulus function was introduced by Nakano (1953). We recall that a modulus f is a function from [0,∞) to [0,∞)such that (i) f(x) =0 if and only if x=0 ,(ii) f(x+y) ≤ f(x)+f(y) for x,y≥0, (iii) f is increasing and (iv) f is continuous from the right at 0. Hence f must be continuous everywhere on [0,∞). Bhardwaj & Dhawan (2015),Kolk (1993), Maddox (1986), Öztürk and Bilgin (1994), Pehlivan and Fisher (1994), Ruckle (1973), and others used a modulus function to construct sequence spaces. Marouf (1993), presented definitions for asymptotically equivalent sequences and asymptotic regular matrices. Patterson (2003), extended these concepts by presenting an asymptotically statistical equivalent analog of these definitions and natural regularity conditions for nonnegative summability matrices . Subsequently, many authors have shown their interest to solve different problems arising in this area (see (Basarir and Altundag,2008). (Basarir and Altundag,2011), (Bilgin,2011), and (Patterson and Savas,2006)). The concept of I-convergence was introduced by Kostyrko et al. (2000/2001) in a metric space. Later it was further studied by Bilgin (2015),,Dass et al. (2011), Dems (2004-2005), Kumar and Sharma (2012), Savaş and Gumus (2013), and many others. In this paper we introduce the concepts asymptotically $\alpha_f^p(I)$ -lacunary equivalence with order α, and asymptotically α_f (I) -lacunary statistical equivalence with order α,which is a natural combination of the defintion for asymptotically equivalent, Ideal convergence, Statistically limit, Lacunary sequence, Modulus function and a sequence of positive real numbers p=(p_k) and also some inclusion theorems are proved.

Materials and Methods

Now we recall some definitions of sequence spaces

Definition 2.1. Two nonnegative sequences [x]and [y] are said to be asymptotically equivalent if : $lim_k \frac{x_k}{y_k}$=1,(denoted by x∽y).

Definition 2.2. Two nonnegative sequences [x] and [y] are said to be asymptotically statistical equivalent of multiple L provided that for every ε > 0, $lim_n \frac{1}{n}|\{\ k\le n: |\frac{x_k}{y_k}-L|\ge\varepsilon\}|=0$,(denoted by $x \overset{S}{\underset{\approx}{}} \backsim y$) and simply asymptotically statistical equivalent, if L=1.

Definition 2.3. Let θ be a lacunary sequence; the two nonnegative sequences [x] and [y] are said to be asymptotically lacunary statistical equivalent of multiple L provided that for every ε > 0,

$lim_r \frac{1}{h_r}|\{ k\in I_r : |\frac{x_k}{y_k}-L|\geq\varepsilon\}|=0$,(denoted by $x \overset{S_\theta}{\approx} y$) and simply asymptotically lacunary statistical equivalent, if L=1.

Definition 2.4. Let f be any modulus; the two nonnegative sequences [x] and [y]are said to be f-asymptotically equivalent of multiple L provided that,

$$lim_k f(|\frac{x_k}{y_k}-L|)=0 \quad \text{(denoted by } x \overset{f}{\approx} y\text{)}$$ and simply strong f-asymptotically equivalent, if L=1.

Definition 2.5. Let f be any modulus, θ=(k_r) be a lacunary sequence, p=(p_k) be a sequence of positive real numbers and 0 < α ≤1; the two nonnegative sequences [x] and [y] are said to be strong α_f^p-asymptotically lacunary equivalent of order α, to multiple L provided that

$$\lim_r \frac{1}{h_r^\alpha}\sum_{k\in I_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k}$$,(denoted by $x \overset{N_\theta,\alpha_f^p}{\approx} y$) and simply strong α_f^p -asymptotically lacunary equivalent, if L=1.

Definition 2.6. Let f be any modulus, θ=(k_r) be a lacunary sequence, and 0 < α ≤1; the two nonnegative sequences [x] and [y] are said to be asymptotically α_f -lacunary statistical equivalent of order α, to multiple L provided that for every ε > 0,

$lim_r \frac{1}{h_r^\alpha}|\{ k\in I_r : f(|\frac{x_k}{y_k}-L|)\geq\varepsilon\}|=0$,(denoted by $x \overset{S_\theta,\alpha_f}{\approx} y$) and simply asymptotically α_f -lacunary statistical equivalent, if L=1.

For any non-empty set X, let P(X) denote the power set of X.

Definition 2.7. A family I⊆P(X)is said to be an ideal in X if

(i) ∅∈I;
(ii) A,B∈I imply A∪B∈I and
(iii) A∈I,B⊂A imply B∈I.

Definition 2.8. A non-empty family F⊆P(X)is said to be a filter in X if

(i) ∅∉F;
(ii) A,B∈F imply A∩B∈F and
(iii) A∈F,B⊃A imply B∈F.

An ideal I is said to be non-trivial if I≠{∅}and X∉I.A non-trivial ideal I is called admissible if it contains all the singleton sets. Moreover, if I is a non-trivial ideal on X, then F=F(I)={X-A:A∈I} is a filter on X and conversely. The filterF(I) is called the filter associated with the ideal I.

Definition 2.9. Let I⊂P(N) be a non-trivial ideal in N and (X,ρ) be a metric space. A sequence [x] in X is said to be I-convergent to ξ if for each ε > 0 , the set {k∈N:ρ(x_k,ξ)≥ε}∈I.

In this case, we write I-$lim_k x_k$=ξ.

Definition 2.10. A sequence [x] of numbers is said to be I-statistical convergent or S(I)-convergent to L, if for every ε > 0 and δ > 0, we have {n∈N;(1/n)|{k≤n:| x_k −L|≥ε}|≥δ}∈I.

In this case, we write x_k→L (S(I)) or S(I) -l$lim_k x_k$=L.

Definition 2.11. Let I⊂P(N) be a non-trivial ideal in N and θ=(k_r) be a lacunary sequence. The two nonnegative sequences [x] and [y] are said to be asymptotically lacunary statistical equivalent of multiple L with respect to the ideal I provided that for each ε > 0 and γ > 0, {r∈N; $\frac{1}{h_r}|\{ k\in I_r : |\frac{x_k}{y_k}-L|\geq\varepsilon\}|\geq\gamma$} ∈ I, denoted by $x \overset{I(S_\theta)}{\approx} y$ and simply asymptotically lacunary statistical equivalent with respect to the ideal I, if L=1.

Definition 2.12. Let I⊂P(N) be a non-trivial ideal in N and θ=(k_r) be a lacunary sequence. The two non-negative sequences [x] and [y] are said to be strongly asymptotically lacunary equivalent of multiple L with respect to the ideal I provided that for ε > 0 {r∈N; $\frac{1}{h_r}\sum_{k\in I_r}|\frac{x_k}{y_k}-L|\geq\varepsilon$} ∈I, denoted by $x \overset{I(N_\theta)}{\approx} y$ and simply strongly asymptotically lacunary equivalent with respect to the ideal I, if L=1.

Definition 2.13. Let I⊂P(N) be a non-trivial ideal in N and f be a modulus function. The two non-negative sequences [x] and [y] are said to be f - asymptotically equivalent of multiple L with respect to the ideal I provided that for each ε > 0 {k∈N; f(|$\frac{x_k}{y_k}$-L|)≥ε } ∈I denoted by $x \overset{I(f)}{\approx} y$ and simply f-asymptotically equivalent with respect to the ideal I, if L=1.

Results and Discussion

We now consider our main results. We begin with the following definitions.

Definition 3.1. Let I⊂P(N) be a non-trivial ideal in N , f be a modulus function, and p = (p_k) be a sequence of positive real numbers and 0 < α ≤1;. Two number sequences [x] and [y] are said to be strongly (f,p)-asymptotically equivalent of multiple L with respect to the ideal I provided that for each ε > 0 ,

$\{n\in N;\ \frac{1}{n^{\alpha}}\sum_{k=1}^{n}[f(|\frac{x_k}{y_k}-L|)]^{p_k}\geq\varepsilon\}\in I$ denoted by $x \overset{w,\alpha_f^p(I)}{\approx} y$ and simply strongly $\alpha_f^p(I)$-asymptotically equivalent with respect to the ideal I, if L=1.

If we take α=1,we write $x \overset{\alpha_f^p(I)}{\approx} y$ instead of $x \overset{w,\alpha_f^p(I)}{\approx} y$.

If we take f(x)=x for x≥0 , we write $x \overset{w,\alpha^p(I)}{\approx} y$ instead of $x \overset{w,\alpha_f^p(I)}{\approx} y$.

If we take p_k=1 for all k∈N, we write $x \overset{w,\alpha_f(I)}{\approx} y$ instead of $x \overset{w,\alpha_f^p(I)}{\approx} y$

Definition 3.2. Let I⊂P(N) be a non-trivial ideal in N , f be a modulus function , θ=(k_r) be a lacunary sequence, and p = $p_k)$ be a sequence of positive real numbers and 0 < α ≤1. Two number sequences [x] and [y] are said to be strongly (f,p)-asymptotically lacunary equivalent of multiple L with respect to the ideal I provided that for each ε > 0,

$\{r\in N;\ \frac{1}{h_r^{\alpha}}\sum_{k\in I_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k}\geq\varepsilon\}\in I$ denoted by $x \overset{N_\theta,\alpha_f^p(I)}{\approx} y$ and simply strongly $\alpha_f^p(I)$-asymptotically lacunary equivalent with respect to the ideal I, if L=1.

If we take α=1 , we write $x \overset{N_\theta,{}_f^p(I)}{\approx} y$ instead of $x \overset{N_\theta,\alpha_f^p(I)}{\approx} y$. Hence $x \overset{N_\theta,{}_f^p(I)}{\approx} y$ is the same as the $x \overset{I(N_\theta^{(f,p)})}{\approx} y$ of Bilgin (2011),

Note that,we take p_k=1 for all k∈N, we write $x \overset{N_\theta,{}_f(I)}{\approx} y$ instead of $x \overset{N_\theta,\alpha_f^p(I)}{\approx} y$. Hence $x \overset{N_\theta,{}_f(I)}{\approx} y$ is the same as the $x \overset{I(N_\theta^f)}{\approx} y$ of Kumar and Sharma (2012). Also if we put f(x)=x for x≥0 , we write $x \overset{N_\theta,{}^p(I)}{\approx} y$ instead of $x \overset{N_\theta,{}_f^p(I)}{\approx} y$. Hence $x \overset{N_\theta,{}^p(I)}{\approx} y$ is the same as the $x \overset{N_\theta^{L(p)}(I)}{\approx} y$ of Savaş and Gumus (2013).

Definition 3.3. Let I⊂P(N) be a non-trivial ideal in N , f be a modulus function , θ=(k_r) be a lacunary sequence, and p=(p_k) be a sequence of positive real numbers and 0<α≤1. the two nonnegative sequences [x] and [y] are said to be asymptotically α_f (I)- -lacunary statistical equivalent of order α, to multiple L with respect to the ideal I provided that for every ε>0,and γ>0,

$\{r\in N;\ \frac{1}{h_r^{\alpha}}|\{\ k\in I_r: f(|\frac{x_k}{y_k}-L|)\geq\varepsilon\}|\geq\gamma\}\in I$ (denoted by $x \overset{S_\theta,\alpha_f(I)}{\approx} y$)

If we take α=1 , we write $x \overset{S_\theta,{}_f(I)}{\approx} y$ instead of $x \overset{S_\theta,\alpha_f(I)}{\approx} y$.

If we put f(x)=x for x≥0, we write $x \overset{S_{\theta,f}(I)}{\approx} y$ instead of $x \overset{S_{\theta},\alpha_f(I)}{\approx} y$.

If we take α=1 , we write $x \overset{S_{\theta}(I)}{\approx} y$ instead of $x \overset{S_{\theta},\alpha(I)}{\approx} y$.

We now prove some inclusion theorems.

Theorem 3.1. Let I⊂P(N) be a non-trivial ideal in N , f be any modulus, θ = (k_r) be a lacunary sequence, p=(p_k) be a sequence of positive real numbers and 0 < α ≤ β ≤1 then

(i) if $x \overset{w,\alpha_f^p(I)}{\approx} y$ implies $x \overset{w,\beta_f^p(I)}{\approx} y$

(ii) if $x \overset{N_{\theta},\alpha_f^p(I)}{\approx} y$ implies $x \overset{N_{\theta},\beta_f^p(I)}{\approx} y$

(iii) if $x \overset{S_{\theta},\alpha_f(I)}{\approx} y$ implies $x \overset{S_{\theta},\beta_f(I)}{\approx} y$.

Proof.(i-iii) Let 0<α≤β≤1 , $x \overset{w,\alpha_f^p(I)}{\approx} y$, $x \overset{N_{\theta},\alpha_f^p(I)}{\approx} y$ and $x \overset{S_{\theta},\alpha_f(I)}{\approx} y$. It is easily that $\frac{1}{n^\beta} \leq \frac{1}{n^\alpha}$ for all n. Since $h_r = k_r - k_{r-1} \to \infty$,we can actually choose r, so that $h_r^\alpha \leq h_r^\beta$ and $\frac{1}{h_r^\beta} \leq \frac{1}{h_r^\alpha}$. Hence

$$\frac{1}{n^\beta}\sum_{k=1}^{n}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \leq \frac{1}{n^\alpha}\sum_{k=1}^{n}[f(|\frac{x_k}{y_k} - L|)]^{p_k}$$

$$\frac{1}{h_r^\beta}\sum_{k\in I_r}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \leq \frac{1}{h_r^\alpha}\sum_{k\in I_r}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \text{ and}$$

$$\frac{1}{h_r^\beta}|\{k\in I_r : f(|\frac{x_k}{y_k} - L|)\geq\varepsilon\}| \leq \frac{1}{h_r^\alpha}|\{k\in I_r : f(|\frac{x_k}{y_k} - L|)\geq\varepsilon\}|$$

Thus,

$$\{n\in N : \frac{1}{n^\beta}\sum_{k=1}^{n}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \geq\varepsilon\} \subseteq \{n\in N ; \frac{1}{n^\alpha}\sum_{k=1}^{n}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \}\in I$$

$$\{k\in I_r ; \frac{1}{h_r^\beta}\sum_{k\in I_r}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \geq\varepsilon\} \subseteq \{k\in I_r ; \frac{1}{h_r^\alpha}\sum_{k\in I_r}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \geq\varepsilon\}\in I$$

$$\{n\in N ; \frac{1}{h_r^\beta}|\{k\in I_r : f(|\frac{x_k}{y_k} - L|)\geq\varepsilon\}|\geq\gamma\} \subseteq \{n\in N ; \frac{1}{h_r^\alpha}|\{k\in I_r : f(|\frac{x_k}{y_k} - L|)\geq\varepsilon\}|\geq\gamma\}\in I.$$

Therefore respecticely $x \overset{w,\beta_f^p(I)}{\approx} y$, $x \overset{N_{\theta},\beta_f^p(I)}{\approx} y$ and $x \overset{S_{\theta},\beta_f(I)}{\approx} y$.

Setting β=1, in Theorem 1 gives the following result .

Corollary. Let f be any modulus, θ=(k_r) be a lacunary sequence, p=(p_k) be a sequence of positive real numbers and 0 < α ≤ 1 , then

(i) if $x \overset{w,\alpha_f^p}{\approx} y$ implies $x \overset{w_f^p(I)}{\approx} y$

(ii) if $x \overset{N_{\theta},\alpha_f^p}{\approx} y$ implies $x \overset{N_{\theta f}^{\,p}(I)}{\approx} y$

(iii) if $x \overset{S_\theta, \alpha_f}{\approx} y$ implies $x \overset{S_{\theta f}(I)}{\approx} y$.

We have this section with the following Theorem to show that the relation between α_f^p(I)- lacunary equivalence with order α and strong α_f^p(I)- lacunary equivalence with order α.

Theorem 3.2. Let I⊂P(N) be a non-trivial ideal in N , f be a modulus function, θ=(k_r) be a lacunary sequence,0<α≤1 and

$0<h=\inf_k p_k \le p_k \le \sup_k p_k = H < \infty$, if $\lim_{t\to\infty} \frac{f(t)}{t} = \beta > 0$, then $x \overset{w,\alpha_f^p(I)}{\approx} y$ implies $x \overset{w,\alpha^p(I)}{\approx} y$

Proof. If $\lim_{t\to\infty} \frac{f(t)}{t} = \beta > 0$, then f(t) ≥βt for all t>0.Let $x \overset{w,\alpha_f^p(I)}{\approx} y$,clearly

$$\frac{1}{n^\alpha}\sum_{k=1}^{n}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \ge \frac{1}{n^\alpha}\sum_{k=1}^{n}[\beta(|\frac{x_k}{y_k} - L|)]^{p_k}$$

$$\ge \min\{\beta^h, \beta^H\}\frac{1}{n^\alpha}\sum_{k=1}^{n}[|\frac{x_k}{y_k} - L|]^{p_k}$$

it follows that for each ε>0, we have

$$\{n\in N; \frac{1}{n^\alpha}\sum_{k=1}^{n}[|\frac{x_k}{y_k} - L|]^{p_k} \ge \varepsilon\} \subseteq \{ n\in N; \frac{1}{n^\alpha}\sum_{k=1}^{n}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \ge \min\{\beta^h, \beta^H\}\ \}\in I.$$

Since $x \overset{w,\alpha_f^p(I)}{\approx} y$, it follows that the later set belongs to I, and therefore, the theorem is proved.

Theorem 3.3. Let I⊂P(N) be a non-trivial ideal in N , f be a modulus function, θ=(k_r) be a lacunary sequence,0<α≤1 and $0<h=\inf_k p_k \le p_k \le \sup_k p_k = H < \infty$, then

if $\lim_{t\to\infty} \frac{f(t)}{t} = \beta > 0$, then $x \overset{N_\theta,\alpha_f^p(I)}{\approx} y$ implies $x \overset{N_\theta,\alpha^p(I)}{\approx} y$.

Proof. The proof of Theorem 3.2 is very similar to the Theorem 3.1.Then we omit it.

The next theorem shows the relationship between the strong α_f^p(I)- lacunary equivalence with order α and the strong α_f^p(I)- equivalence with order α.

Theorem 3.4. Let I⊂P(N) be a non-trivial ideal in N , f be a modulus function, θ=(k_r) be a lacunary sequence,0<α≤1 and p = (p_k) be a sequence of positive real numbers , then

(i) if $\sup_r \frac{1}{k_{r-1}^\alpha}\sum_{m=1}^{r}(k_m - k_{m-1})^\alpha$ =B(say)<∞ then $x \overset{N_\theta,\alpha_f^p(I)}{\approx} y$ implies $x \overset{w,\alpha_f^p(I)}{\approx} y$

(ii) if $\sup_r \frac{k_r}{h_r^\alpha}$ =C(say)<∞ then $x \overset{w,\alpha_f^p(I)}{\approx} y$ implies $x \overset{N_\theta,\alpha_f^p(I)}{\approx} y$.

Proof.(i). Now suppose that $x \overset{N_\theta,\alpha_f^p(I)}{\approx} y$ and ε>0.Let $A=\{r\in N; \frac{1}{h_r^\alpha}\sum_{k\in I_r}[f(|\frac{x_k}{y_k} - L|)]^{p_k} < \varepsilon\}$,

Hence,for all j∈A, we have $H_j = \frac{1}{h_j^\alpha}\sum_{k\in I_j}[f(|\frac{x_k}{y_k} - L|)]^{p_k} < \varepsilon$. Choose n is any integer with $k_r \ge n > k_{r-1}$} where r∈A. Now write

$$\frac{1}{n^\alpha}\sum_{k=1}^{n}[f(|\frac{x_k}{y_k} - L|)]^{p_k} \le \frac{1}{k_{r-1}^\alpha}\sum_{k=1}^{k_r}[f(|\frac{x_k}{y_k} - L|)]^{p_k}$$

$$\leq B\frac{1}{h_m^{\alpha}}\sum_{k\in I_m}[f(|\frac{x_k}{y_k}-L|)]^{p_k}$$

$$\{n\in N;\ \frac{1}{n^{\alpha}}\sum_{k=1}^{n}[f(|\frac{x_k}{y_k}-L|)]^{p_k}\ \geq\varepsilon\}\subseteq\{r\in N;\ \frac{1}{h_r^{\alpha}}\sum_{k\in I_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k}\ \geq\varepsilon/B\}\in I.$$

which yields that $x \overset{w,\alpha_f^p(I)}{\approx} y$.

(ii). Let $x \overset{w,\alpha_f^p(I)}{\approx} y$.

$$\frac{1}{h_r^{\alpha}}\sum_{k\in I_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k} = \frac{1}{h_r^{\alpha}}\sum_{k=1}^{k_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k} - \frac{1}{h_r^{\alpha}}\sum_{k=1}^{k_{r-1}}[f(|\frac{x_k}{y_k}-L|)]^{p_k}$$

$$\leq \frac{1}{h_r^{\alpha}}\sum_{k=1}^{k_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k}$$

$$= \frac{k_r}{h_r^{\alpha}k_r}\sum_{k=1}^{k_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k}$$

$$< C\frac{1}{k_r}\sum_{k=1}^{k_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k}$$

$$\{r\in N;\ \frac{1}{h_r^{\alpha}}\sum_{k=1}^{k_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k}\ \geq\varepsilon\}\subseteq\{r\in N;\ \frac{1}{k_r}\sum_{k=1}^{k_r}[f(|\frac{x_k}{y_k}-L|)]^{p_k}\ \geq\varepsilon/C\}\in I.$$

which yields that $x \overset{N_{\theta},\alpha_f^p(I)}{\approx} y$.

Now we give relation between asymptotically α- lacunary statistical equivalence and strong α_f^p (I)- lacunary equivalence with order α.Also we give relation between asymptotically α_f - lacunary statistical equivalence and strong α_f^p(I)- lacunary equivalence with order α.The Proofs will not be given.

Theorem 3.5. Let I⊂P(N) be a non-trivial ideal in N , f be a modulus function, θ=(k_r) be a lacunary sequence,0<α≤1 and $0<h=\inf_k p_k \leq p_k \leq \sup_k p_k = H<\infty$, then $x \overset{N_{\theta},\alpha_f^p(I)}{\approx} y$ implies $x \overset{S_{\theta},\alpha(I)}{\approx} y$.

Theorem 3.6. Let I⊂P(N) be a non-trivial ideal in N , f be a modulus function, θ=(k_r) be a lacunary sequence,0<α≤1 and $0<h=\inf_k p_k \leq p_k \leq \sup_k p_k = H<\infty$, then $x \overset{N_{\theta},\alpha_f^p(I)}{\approx} y$ implies $x \overset{S_{\theta},\alpha_f(I)}{\approx} y$,

Let p_k=p for all k , t_k=t for all k and 0<p≤t. Then it follows following Theorem.

Theorem 3.7. Let f be any modulus, θ=(k_r) be a lacunary sequence, p=(p_k) be a sequence of positive real numbers,and 0 < α ≤ 1 then $x \overset{N_{\theta},\alpha_f^t(I)}{\approx} y$ implies $x \overset{N_{\theta},\alpha_f^p(I)}{\approx} y$,

Conclusion

The relations we have achieved are generally parallel to the literature. However, some of the relations in the literature have not been found

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