BOHMIAN MECHANICS MADE MACHIAN

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Abstract: Non-relativistic shape dynamics is a Machian theory of particle interactions. It comes with three constraints to be satisfied if the Newtonian absolute space is to be used. On the other hand, Bohmian mechanics is a realistic interpretation of quantum mechanics. It is a type of non-local hidden variables theory. We supplemented Bohmian mechanics with appropriately calculated shape dynamics constraints to make it fully Machian.

Keywords: Bohmian mechanics, Mach's principle, Shape dynamics

Introduction

There are many versions of the Mach's Principle in the literature. Bondi and Samuel (1997) cite about ten versions of this principle. The version we consider is Julian Barbour's interpretation (Barbour, 2010). He states that in order for a theory to be Machian, a tangent vector or a direction in the reduced configuration (configuration space quotiented by the gauge groups) space should be able to determine the evolution of the system uniquely (Barbour, 2010).

Bohmian Mechanics (BM) is a theory of quantum mechanics without observers (Bricmont, Dürr, Galavotti, Ghirardi, Petruccione & Zanghi, 2001; Bell 2004; Golstein 1998a; Goldstein 1998b). It is a nonlocal hidden variables theory. The hidden variables are the exact positions of each particle. There are two main concepts: The wavefunction, and the particle positions. Wavefunction evolves according to the Schrödinger equation (Bricmont et al, 2001, p. 115):

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi. \tag{1}$$

On the other hand, the velocity of particles are guided by the wavefunction using the probability current (Bricmont et al, 2001, p. 115):

$$\vec{v}_k = \frac{\hbar}{m_k} \frac{\Im\left(\Psi^* \vec{\nabla}_k \Psi\right)}{\Psi^* \Psi}, \tag{2}$$

where k runs in particle numbers and $\Im(\cdot)$stands for the imaginary part of the term inside the parenthesis. Be aware that the Equation (2) is a first order differential equation. Therefore, the particle paths never cross each other.

Shape Dynamics (SD) is first initiated by Barbour where a simple introduction is given in (Barbour, 2012) and then made compatible with Einstein's general theory of gravitation (GR) by including other researchers (Barbour, & Murchadha, 1999; Anderson, Barbour, Foster, & Murchadha, 2003; Anderson, Barbour, Foster, Kelleher, & Murchadha, 2005; Gomes, Gryb, & Koslowski, 2011; Gomes, & Koslowski, 2012). Readers may see (Mercati, 2014) for a review of SD. The fully developed SD has the same reduced configuration space as GR but possesses a different gauge group. The gauge group of SD is local scale (Weyl) symmetry whereas in GR we have local Lorentz symmetry. SD is a fully Machian theory. The version of SD we consider in this paper is Julian Barbour's non-relativistic version (Barbour, 2012). The reason we use this version of SD is that BM is also non-relativistic.

SD considers inner-particle separations and and their time derivatives as the true relational variables. In an N-particle system there are $N(N-1)/2$ many inter-particle separations. If we move on to Newtonian absolute space there are $3N$ many variables in a 3D space. For that reason, we will use the Newtonian absolute space. In order to make a theory Machian, SD comes with various constraints in Newtonian absolute space. Therefore, Newton's absolute space can be used as long as the following constraints are satisfied and the theory will be Machian regardless (we modified the constraints appearing in (Barbour, 2012) according to quantum mechanics):

$$H\Psi = 0, \tag{3}$$
$$\sum_k \vec{L}_k \Psi = 0 \tag{4}$$
$$\sum_k \vec{r}_k \cdot \vec{p}_k \Psi = 0 \tag{5}$$

where $\vec{L}_k = \vec{r}_k \times \vec{p}_k$ and $\vec{p}_k = -i\hbar\vec{\nabla}_k$ as usual. The first is the energy constraints, the second is the angular momentum constraint and finally the third is the dilational momentum constraint.

The Machian Description of Bohmian Mechanics

In this section we will modify the SD constraints according to the BM variables and make BM Machian. In BM, we already have exact velocity of a particle. By multiplying it with the mass of the particle, one can find the exact momentum of that particle. Therefore, the use of momentum operator, $-i\hbar\vec{\nabla}_k$, is unnecessary. One may argue against our argument by saying that "While there is momentum operator in quantum mechanics, why do you not use it?" Our answer would be that the momentum operator in quantum mechanics is about making measurements. Its expectation value is the expected value of momentum. However, in BM we have the exact momenta of particles. Hence, it is natural to use them. As a note, please pay attention to the fact that momenta in BM are functions of the wavefunction, see Equation (2). The wavefunction still plays a central role in our description.

In a similar way, we can write down the orbital angular momentum as $m_k\vec{r}_k \times \vec{v}_k$ instead of $\vec{L}_k\Psi$. The Hamiltonian is kinetic energy plus the potential energy. The Hamiltonian constraint can be wrote down as follows instead of $H\Psi = 0$:

$$H_{BM} = \sum_k \frac{1}{2} m_k v_k^2 + \frac{1}{2}\sum_{k,k'} V(\vec{r}_k, \vec{r}_{k'}) = 0. \tag{6}$$

We added a subscript BM to H in order to mark that it is calculated using the particle positions. All in all, the equations of motion in BM remains the same:

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi. \tag{7}$$
$$\vec{v}_k = \frac{\hbar}{m_k}\frac{\Im\left(\Psi^*\vec{\nabla}_k\Psi\right)}{\Psi^*\Psi}, \tag{8}$$

where H is the usual Hamiltonian from the quantum mechanics. The constraints on the other hand are as follows:

$$H_{BM} = \sum_k \frac{1}{2} m_k v_k^2 + \frac{1}{2}\sum_{k,k'} V(\vec{r}_k, \vec{r}_{k'}) = 0, \tag{9}$$
$$\vec{L}_{BM} = \sum_k m_k\vec{r}_k \times \vec{v}_k = 0, \tag{10}$$
$$D_{BM} = \sum_k m_k\vec{r}_k \cdot \vec{v}_k. \tag{11}$$

Machian BM solutions are those that satisfy the equations of motion and the constraints.

Conclusion

In this paper, we introduced Bohmian mechanics and shape dynamics. Bohmian mechanics has been made Machian by supplementing its equations of motion by appropriate shape dynamics constraints. It is no surprise that Bohmian mechanics uses Newtonian absolute space as it is the case with the orthodox quantum mechanics. However, the use of absolute space by Bohmian mechanics is allowed if the solutions respect the shape dynamics constraints and the theory is Machian when supplemented with these constraints.

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References

Anderson, E., Barbour, J., Foster, B., & Murchadha, N. O. (2003). Scale-invariant gravity: Geometrodynamics. *Classical and Quantum Gravity*, 20(8), 1571.

Anderson, E., Barbour, J., Foster, B. Z., Kelleher, B., & Murchadha, N. Ó. (2005). The physical gravitational degrees of freedom. *Classical and quantum gravity*, 22(9), 1795.

Barbour, J. (2012). Shape dynamics. An introduction. In *Quantum field theory and gravity* (pp. 257-297). Springer, Basel.

Barbour, J., & Murchadha, N. O. (1999). Classical and quantum gravity on conformal superspace. *arXiv preprint gr-qc/9911071*.

Barbour, J. (2010). The definition of Mach's principle. *Foundations of Physics*, *40*(9-10), 1263-1284.

Bell, J. S. (2004). *Speakable and unspeakable in quantum mechanics: Collected papers on quantum philosophy*. Cambridge university press.

Bondi, H., & Samuel, J. (1997). The Lense-Thirring effect and Mach's principle. *Physics Letters A*, *228*(3), 121-126.

Bricmont, J., Dürr, D., Galavotti, M. C., Ghirardi, G., Petruccione, F., & Zanghi, N. (2001). Chance in physics. In *Chance in Physics, Edited by J. Bricmont, D. Dürr, MC Galavotti, G. Ghirardi, F. Petruccione, N. Zanghi, Lecture Notes in Physics, vol. 574*.

Goldstein, S. (1998a). Quantum mechanics without observers. *Physics Today*, *51*(3) 42-47.

Goldstein, S. (1998b). Quantum theory without observers--Part two. *Physics Today*, *51*(4), 38-42.

Gomes, H., Gryb, S., & Koslowski, T. (2011). Einstein gravity as a 3D conformally invariant theory. *Classical and Quantum Gravity*, *28*(4), 045005.

Gomes, H., & Koslowski, T. (2012). The link between general relativity and shape dynamics. *Classical and Quantum Gravity*, *29*(7), 075009.

Mercati, F. (2014). A shape dynamics tutorial. *arXiv preprint arXiv:1409.0105*.

